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Quantum gravity

Research article

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Abstract

Background:

Incorporating both the principles of quantum theory and the principles of relativity theory into one unique mathematical framework could be of help to provide us with a satisfactory description of the microstructure of spacetime, even at the so-called Planck scale, while unifying the four basic fields of nature.

Methods:

Our contemporary understanding of gravity is based more or less on general relativity theory, which prefers to describe gravitation from a geometric point of view as something like curvature of spacetime caused by matter and energy. Therefore, the quantization of gravity as derived in this publication is based on the quantization of spacetime geometry independently of extreme technical difficulties and the profound methodological and other challenges. Furthermore, the treatment of time is of central importance in any form of quantum gravity. The relationship between time and gravitational field has been reinvestigated again.

Results:

A relativistic, gravitational Schrödinger wave equation has been derived. A re-examination of the relationship between time and gravitational field implies the equivalence of time and gravitational field.

Conclusion:

The gravitational field itself has been quantized.

Keywords: Cause; Effect; Causation; Energy; Time; Space; Quantum gravity

1. Introduction

It is not easy to say what energy, time and space [1], [2] really are. Nonetheless, a new focus on these old questions might generate at least some new arguments and insights. In compliance with the time-honoured principle of going **from the known to the unknown**, it seems reasonable to highlight quite a few positions of single authors. One of these single authors is Tycho Brahe (1546-1601), born

[1]Ashtekar A. New variables for classical and quantum gravity. Phys Rev Lett. 1986 Nov 3;57(18):2244-2247. doi: 10.1103/PhysRevLett.57.2244. PMID: 10033673.

[2]Rovelli C. Loop Quantum Gravity. Living Rev Relativ. 2008;11(1):5. doi: 10.12942/lrr-2008-5. Epub 2008 Jul 15. PMID: 28179822; PMCID: PMC5256093.

Tyge Ottesen Brahe (Brahe, 1602), a Danish astronomer, who performed very accurate astronomical observations of the motions of planets and their moons. Based on an analysis of the data obtained by Brahe's own observations of objective reality (Einstein et al., 1935a), Brahe's appointed successor at the court of Rudolph II, the German astronomer Johannes Kepler (1571-1630), derived his three laws of planetary motion (Kepler, 1609). Fortunately, it didn't take that long, and Isaac Newton (1643-1727) put forward his famous law of gravitation (Newton, 1687) in 1687. However, it deserves to be considered that especially Newton himself insisted on absolute space and absolute time. Nonetheless, in the history of physics from Aristotle through to Leibniz and other, authors denied that space and time (see Barukčić, 2011) are real entities at all. Leibniz himself simply denied any mind-independent reality of space and time. To make matters worse, the reader should note that as long as we have to follow, be it voluntary or unvoluntary, the path laid out by the very influential idealistic German philosopher Immanuel Kant (1724–1804), we do face the fact that

> "Time is not
>
> something objective and real,
>
> neither a substance, nor an accident, nor a relation. "
>
> (see Kant, 1770)
>
> (see also (English) Kant, 1894, p. 61)

Kant himself from his own perspective leaves no scope for any reasonable doubt about space to.

> "Space is not
>
> something objective and real,
>
> neither substance, nor accident, nor relation ;
>
> but subjective and ideal ... "
>
> (see also Kant, 1770)
>
> (see also (English) Kant, 1894, p. 65)

The German philosopher Immanuel Kant (see also Kant, Immanuel, 1786) published 1786 his book Metaphysical Foundations of Natural Science (German: Metaphysische Anfangsgründe der Naturwissenschaft) which had a basic influence on scientific development. Kant tried to show that human knowledge can be gained independently of any human experience or experiment. At the end, categories or pure concepts of the understanding are enough. However, this concept did not go unchallenged. Let us hear what Albert Einstein thinks about Kant's metaphysics and the importance of Kant in the foundations of natural science.

"Das ist die Erbsünde Kants, dass Begriffe und Kategorien, die nicht aus der Erfahrung gewonnen werden können, zum Verständnis dieser Erfahrung notwendig sein sollen. Unbefriedigend bleibt dabei aber immer die Willkür der Auswahl derjenigen Elemente, die man als apriorisch bezeichnet. "

(see also Fölsing, Albrecht, 1993, p. 544)

Furthermore, with respect to Kant's point of view, Albert Einstein (1879–1955) is asking in general:

"What, then, impels us to devise theory after theory?

Why do we devise theories at all? ...

Because we enjoy ... reducing phenomena by the process of logic

to something already known or (apparently) evident. "

(see also Einstein, 1950, p. 13)

Newton's theory of gravitation, space and time collided soon with Albert Einstein's (1879–1955) point of view and Einstein's special (see Einstein, 1905c,d, 1908) and general (see Einstein, 1915, 1916, 1917, 1950, Einstein and de Sitter, 1932) theory of relativity. Einstein's mathematical unification of space and time into space-time, first proposed by the mathematician Hermann Minkowski (1864-1909) in 1908 (Minkowski, 1908, 1909), culminated in Einstein's colossal intellectual jump, the theory of general relativity which makes use of a more or less four-dimensional non-Euclidean continuum (space-time) while the curvature of space-time itself is determined by the distribution of energy/matter. Gravitation is to a greater or lesser extent geometrized and rather a manifestation of the curvature of space-time than a force. In particular, the very strong and equally one weak spot of Einstein's general theory of relativity is the geometrization of the gravitational field because other fields like the electromagnetic field and the stress energy tensor of matter are thus far devoid (see Goenner, 2004) of any geometrical significance. No sooner said than done, Einstein himself tried to overcome this far-reaching shortcoming of his field equations and stressed the need to go beyond himself by advocating the necessity of "*the establishment of a unified theory by a generalization of the relativistic theory of gravitation*." (see Einstein, 1945, p. 578). In the light of what just has been emphasized and in order to achieve something like a unified field theory it is necessary and appropriate to geometrize (see Einstein and Bergmann, 1938, Einstein, 1925, 1945, 1950) the electromagnetic field and the stress energy tensor of matter too, something much easier said than done. Nonetheless, against all odds and difficulties, the unified field theory, a "*... theory we are looking for must therefore be a generalization of the theory of the gravitational field.*" (see Einstein, 1950, p. 16) It is obviously apparent that all this raises at least the question, what is space-time at all, is it something real? Are the past space-times as real as the present one? Can we identify the deep trace of the past in space-time, like the trace of bare feet in the grainy sand?

2. Material and methods

2.2. Definitions

Definitions have interested scientist since ancient times, while several types of definitions are often in play. However, the character of a definition might vary with its own function. In point of fact, inappropriate definitions can cause a lot of considerable damage.

2.3. Numbers and anti-numbers

Definition 2.1 (Numbers and anti-numbers). *In general, let $E({}_{R}U_{t})$ denote a number from the point of view of a stationary observer(Einstein, 1905d), let $E({}_{R}\underline{U}_{t})$ denote an anti number, the other of the number $E({}_{R}U_{t})$ again from the point of view of a stationary observer(Einstein, 1905d). In general, it is*

$$ {}_{R}U_{t} \equiv E({}_{R}U_{t}) + E({}_{R}\underline{U}_{t}) \tag{1} $$

In particular, ${}_{R}U_{t}$ is in the state of superposition, a law which has been re-formulated by the Danish geologist Nicolaus Steno (see Stenonis, Nicolai, 1669) in his 1696 book 'De Solido Intra Naturaliter Contento Dissertationis Prodomus '. Thus far, under conditions where ${}_{R}U_{t}$ = +0, it is

$$ +E({}_{R}U_{t}) \equiv -E({}_{R}\underline{U}_{t}) \tag{2} $$

and vice versa. It is equally possible that

$$ -E({}_{R}U_{t}) \equiv +E({}_{R}\underline{U}_{t}) \tag{3} $$

Example.

Let $E({}_{R}U_{t}) \equiv +2$, let $E({}_{R}\underline{U}_{t}) \equiv +3$, then ${}_{R}U_{t} \equiv E({}_{R}U_{t}) + E({}_{R}\underline{U}_{t}) \equiv +2+3 \equiv +5$. The concept of numbers and anti-numbers represents an attempt to work out the foundations for unifying geometry or classical logic with number theory, algebra and probability theory. In today's mathematics, probabilistic number theory founded by Paul Erdös, Aurel Friedrich Wintner and Mark Kac during the 1930s [3], [4] is a subfield of number theory, which explicitly deals about probability theory. The probability of a certain number would be $p(E({}_{R}U_{t}) \equiv {}_{R}U_{t}) \equiv p(2 \equiv 5) \equiv \frac{E({}_{R}U_{t})}{{}_{R}U_{t}} \equiv \frac{+2}{+5} \equiv 0.4$. Under these conditions, the probability of a number +2 would describe the extent to which the number +2 is the determining part of the number +5. Figure 1 might illustrate the basic relationship between number theory, algebra and geometry in terms of Euclid's theorem in more detail.

[3] Paul Erdös and Mark Kac (1939, April). On the Gaussian law of errors in the theory of additive functions. In Proceedings of the National Academy of Science (Vol. 25, No. 4, pp. 206-207). https://doi.org/10.1073/pnas.25.4.206

[4] Paul Erdös and Aurel Friedrich Wintner (1939). Additive arithmetical functions and statistical independence. American Journal of Mathematics, 61(3), 713-721. https://doi.org/10.2307/2371326

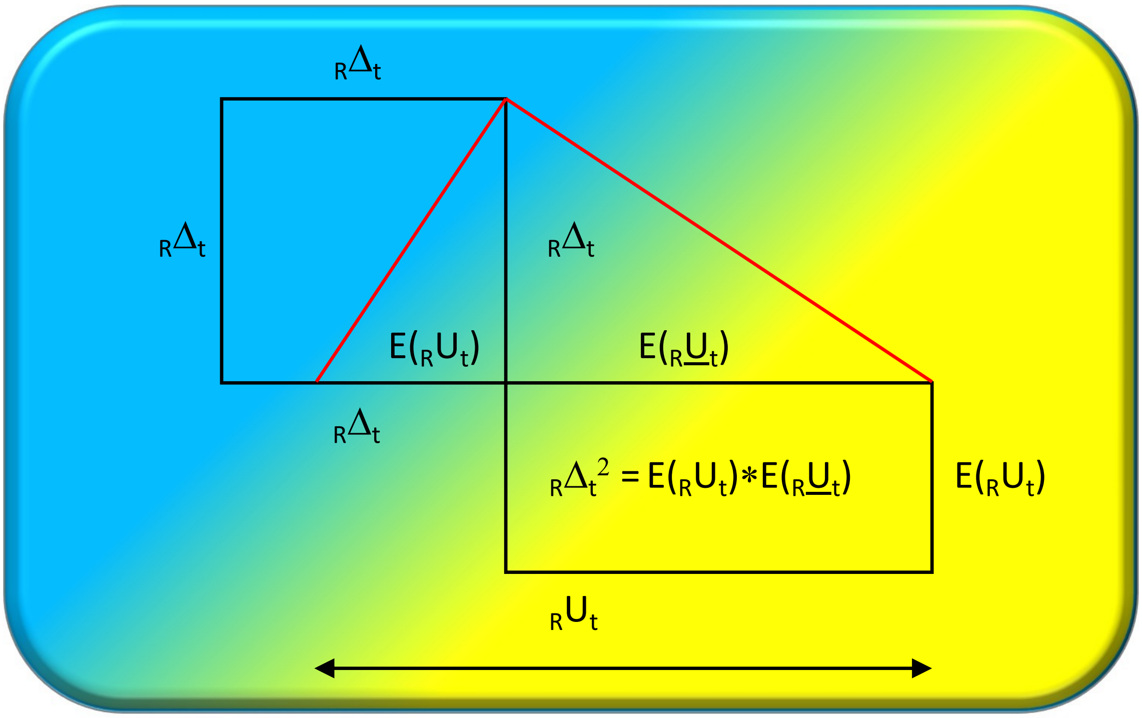

Figure 1. Euclid's theorem, number theory and geometry.

Multiplying equation 1 by the term $_RU_t$, we obtain

$$({_RU_t} \times {_RU_t}) \equiv (E({_RU_t}) \times {_RU_t}) + (E({_R\underline{U}_t}) \times {_RU_t}) \tag{4}$$

We define in general

$$_RU_t{}^2 \equiv {_RC_t}^2 \equiv ({_RU_t} \times {_RU_t}) \tag{5}$$

and

$$E({_RU_t}^2) \equiv {_Ra_t}^2 \equiv (E({_RU_t}) \times {_RU_t}) \tag{6}$$

and

$$E({_R\underline{U}_t}^2) \equiv {_Rb_t}^2 \equiv (E({_R\underline{U}_t}) \times {_RU_t}) \tag{7}$$

Figure 2 might illustrate these relationships in more detail. The Pythagorean theorem as the foundation of a relativistic number theory is given as

$$_Ra_t{}^2 + {_Rb_t}^2 \equiv {_RC_t}^2 \tag{8}$$

It is possible to normalize the relationship of equation 1. We obtain

$$\left(\frac{E({_RU_t})}{_RU_t}\right) + \left(\frac{E({_R\underline{U}_t})}{_RU_t}\right) \equiv \frac{_RU_t}{_RU_t} \equiv +1 \tag{9}$$

where $p(_RU_t) \equiv \left(\frac{E(_RU_t)}{_RU_t}\right)$ *is the probability of the number* $_RU_t$*, and* $p(_R\underline{U}_t) \equiv \left(\frac{E(_R\underline{U}_t)}{_RU_t}\right)$ *is the probability of the anti number* $_R\underline{U}_t$*. A number* $E(_RU_t)$ *is equally a negation of its own anti number* $E(_R\underline{U}_t)$*. In general, it is*

$$E(_RU_t) \equiv {_RU_t} - E(_R\underline{U}_t) \tag{10}$$

We define in general negation as

$$\neg \equiv {_RU_t} - \tag{11}$$

Equation 10 changes to

$$E(_RU_t) \equiv {_RU_t} - E(_R\underline{U}_t) \equiv \neg E(_R\underline{U}_t) \tag{12}$$

However, an anti number itself is a negation. In last consequence, a number is the negation of negation.

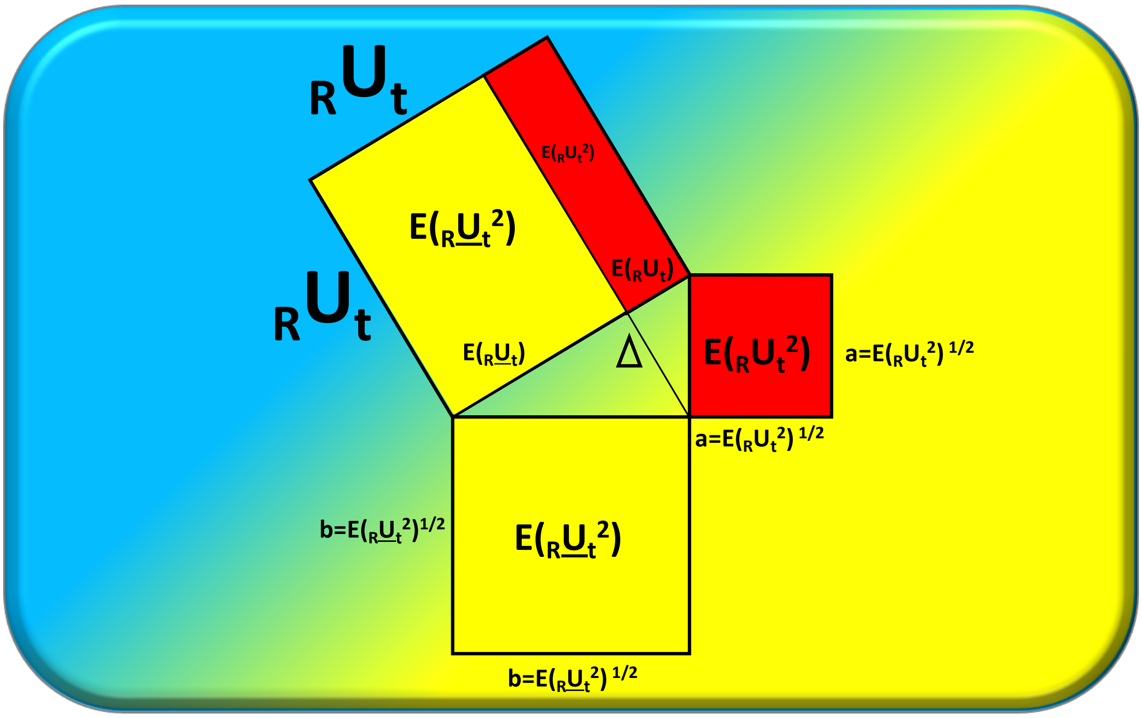

Figure 2. Pythagorean theorem as the foundation of a relativistic number theory.

Equation 9 changes to

$$E(_RU_t) \equiv {_RU_t} \times \left(+1 - \left(\frac{E(_R\underline{U}_t)}{_RU_t}\right)\right) \tag{13}$$

The number $E(_RU_t)$ *is given by the relationship*

$$E(_RU_t)^{+2} \equiv \left({_RU_t}^{+2}\right) \times \left(+1 - \left(\frac{\sigma(_RU_t)^{+2}}{({_RU_t}^{+2})}\right)\right) \tag{14}$$

where $\sigma\left({}_{R}U_{t}\right)^{+2}$ *is the variance and is given by*

$$\sigma\left({}_{R}U_{t}\right)^{+2} \equiv \Delta^{+2} \equiv E\left({}_{R}U_{t}{}^{+2}\right) - E\left({}_{R}U_{t}\right)^{+2} \equiv E\left({}_{R}U_{t}\right) \times E\left({}_{R}\underline{U}_{t}\right) \equiv \left({}_{R}U_{t}{}^{+2}\right) \times p\left({}_{R}U_{t}\right) \times \left(1 - p\left({}_{R}U_{t}\right)\right) \tag{15}$$

2.4. Probability theory

Probability theory, historically an intellectual latecomer, plays more and more an important role in almost all the sciences and finds its way even into philosophy. Noteworthy is that non-negativity and normalization axioms of probability theory are largely matters of convention. Still, probabilities lie between +0 and +1, inclusive. Weighing all the circumstances of an individual event, negative probabilities are theoretically possible. Nonetheless, to be sure, does nothing else remain for us but to accept solely and exclusively today's probability's mathematical treatment? More formally, does geometry (see also Barukčić, 2021) really has nothing to say about probability? What is probability is a question everyone needs to find an answer for themselves?

2.4.1. The probability of an event ${}_{\mathrm{R}}\mathrm{E}_{\mathrm{t}}$

Definition 2.2 **(The probability of an event $\mathbf{{}_{R}E_{t}}$).** *In general, let ${}_{R}E_{t}$ denote an event from the point of view of a stationary observer(Einstein, 1905d), let p(${}_{R}E_{t}$) represent the probability of this event (i.e. energy content) ${}_{R}E_{t}$ at Bernoulli trial t, let E(${}_{R}E_{t}$) represent the expectation value of this event at Bernoulli trial t. Let $\Psi\left({}_{R}E_{t}\right)$ represent the wave function, a probability amplitude (Born, 1926) of an event i.e. ${}_{R}E_{t}$ at a given (period of) time / Bernoulli trial (Uspensky, 1937) t. Let $\Psi^{*}\left({}_{R}E_{t}\right)$ denote the complex conjugate of a wave function. In general, the probability of an event(see Scheid, 1992, p. 72) is determined as*

$$
\begin{aligned}
p\left({}_{R}E_{t}\right) &\equiv p\left({}_{R}E_{t}\right)\\
&\equiv p\left({}_{R}E_{t}\right)\times(+1)\\
&\equiv p\left({}_{R}E_{t}\right)\times\frac{\left({}_{R}E_{t}\right)}{\left({}_{R}E_{t}\right)}\\
&\equiv \frac{p\left({}_{R}E_{t}\right)\times\left({}_{R}E_{t}\right)}{\left({}_{R}E_{t}\right)}\\
&\equiv \frac{E\left({}_{R}E_{t}\right)}{{}_{R}E_{t}}\\
&\equiv p\left({}_{R}E_{t}\right)\times\frac{p\left({}_{R}E_{t}\right)\times\left({}_{R}E_{t}\times{}_{R}E_{t}\right)}{p\left({}_{R}E_{t}\right)\times\left({}_{R}E_{t}\times{}_{R}E_{t}\right)}\\
&\equiv p\left({}_{R}E_{t}\right)\times\frac{\left({}_{R}E_{t}\times{}_{R}E_{t}\right)}{\left({}_{R}E_{t}\times{}_{R}E_{t}\right)}\equiv\frac{E\left({}_{R}E_{t}{}^{2}\right)}{\left({}_{R}E_{t}\times{}_{R}E_{t}\right)}\\
&\equiv \frac{p\left({}_{R}E_{t}\right)\times p\left({}_{R}E_{t}\right)\times\left({}_{R}E_{t}\times{}_{R}E_{t}\right)}{p\left({}_{R}E_{t}\right)\times\left({}_{R}E_{t}\times{}_{R}E_{t}\right)}\\
&\equiv \frac{E\left({}_{R}E_{t}\right)^{2}}{E\left({}_{R}E_{t}{}^{2}\right)}\\
&\equiv \Psi({}_{R}E_{t})\times\Psi^{*}({}_{R}E_{t})
\end{aligned}
\tag{16}
$$

In general, it is,

$$
\Psi^{*}({}_{R}E_{t})\equiv\frac{p({}_{R}E_{t})}{\Psi({}_{R}E_{t})}
\tag{17}
$$

and equally (see equation 15)

$$
ln\left(E\left({}_{R}U_{t}\right)\times E\left({}_{R}\underline{U}_{t}\right)\right)\equiv ln\left(E\left({}_{R}U_{t}\right)\right)+ln\left(E\left({}_{R}\underline{U}_{t}\right)\right)\equiv ln\left(\sigma\left({}_{R}U_{t}\right)^{2}\right)
\tag{18}
$$

However, under conditions, where $p({}_{R}E_{t})=+1$ it is

$$
\Psi^{*}({}_{R}E_{t})\equiv\frac{+1}{\Psi({}_{R}E_{t})}
\tag{19}
$$

By hook or by crook, the term $\frac{+1}{\Psi({}_{R}E_{t})}$ is one essential and determining part of the complex conjugate term $\Psi^{*}({}_{R}E_{t})$. Under these conditions $(p({}_{R}E_{t})=+1)$, it is equally

$$
\Psi({}_{R}E_{t})\times\Psi^{*}({}_{R}E_{t})\equiv+1
\tag{20}
$$

2.4.2. Wave function tensor

At the same time, we strive to define a wave-function tensor denoted by $\Psi_{\mu\nu\ldots}$.

Definition 2.3 (**Wave-function tensor** $\Psi_{\mu\nu\ldots}$).

$$\Psi_{\mu\nu\ldots} \equiv \Psi \times g_{\mu\nu\ldots} \tag{21}$$

or

$$\Psi^{\mu\nu\ldots} \equiv \Psi \times g^{\mu\nu\ldots} \tag{22}$$

et cetera.

2.4.3. Complex conjugate Wave function tensor

The complex conjugate wave-function tensor denoted by ${}^{*}\Psi_{\mu\nu\ldots}$ is defined as described below.

Definition 2.4 (**Complex conjugate Wave-function tensor** ${}^{*}\Psi_{\mu\nu\ldots}$).

$${}^{*}\Psi_{\mu\nu\ldots} \equiv \Psi^{*} \times g_{\mu\nu\ldots} \tag{23}$$

or

$${}^{*}\Psi^{\mu\nu\ldots} \equiv \Psi^{*} \times g^{\mu\nu\ldots} \tag{24}$$

et cetera.

2.4.4. Normalization condition

The inverse metric tensor $g^{\mu\nu}$ is of the same size as the metric tensor $g_{\mu\nu}$. Thus far, whatever $g_{\mu\nu}$ does, $g^{\mu\nu}$ undoes. Under certain circumstances, their product is the identity or unity tensor $1_{\mu\nu}$.

$$g_{\mu\nu\ldots} \cap g^{\mu\nu\ldots} \equiv 1_{\mu\nu\ldots} \tag{25}$$

where $\cap$ denote the commutative multiplication of tensors. In the following, equation 25 is multiplied by $\Psi({}_{\mathrm{R}}E_{\mathrm{t}}) \times \Psi^{*}({}_{\mathrm{R}}E_{\mathrm{t}})$. We obtain

$$g_{\mu\nu\ldots} \cap g^{\mu\nu\ldots} \times \Psi \times \Psi^{*} \equiv 1_{\mu\nu\ldots} \times \Psi \times \Psi^{*} \equiv (g_{\mu\nu\ldots} \times \Psi) \cap (g^{\mu\nu\ldots} \times \Psi^{*}) \equiv \Psi_{\mu\nu\ldots} \times \Psi^{*\mu\nu\ldots} \tag{26}$$

There are conditions where,

$$\Psi_{\mu\nu\ldots} \times \Psi^{*\mu\nu\ldots} \equiv 1_{\mu\nu\ldots} \tag{27}$$

However, this need not be given in general.

2.5. Geometry

Geometry can be traced back to the first trials of systematic logical thinking of humans. Still, the nature of the relation between the definitions, axioms, theorems, and proofs in a system of geometry and objective reality has to be considered in detail. Tensors are one mathematical approach to geometry.

The tensor (see also Voigt, 1898, p. 20) calculus has been developed in some greater detail by Ricci-Curbastro (see Ricci-Curbastro and Levi-Civita, 1900) and his student Levi-Civita on the basis of earlier work of authors like Riemann, Christoffel, Bianchi and others. Especially, Einstein's general theory of relativity is expressed by the mathematical technology of tensors.

2.5.1. Tensor addition

Definition 2.5 (Tensor addition).

The sum of two second rank co-variant (Sylvester, 1851) tensors has the properties of associativity and commutativity and is defined as

$$\begin{aligned} C_{\mu\nu} &\equiv A_{\mu\nu} + B_{\mu\nu} \\ &\equiv B_{\mu\nu} + A_{\mu\nu} \end{aligned} \tag{28}$$

The sum of two second rank contra-variant tensors has the properties of associativity and commutativity and is defined as

$$\begin{aligned} C^{\mu\nu} &\equiv A^{\mu\nu} + B^{\mu\nu} \\ &\equiv B^{\mu\nu} + A^{\mu\nu} \end{aligned} \tag{29}$$

The sum of two second rank mixed tensors has the properties of associativity and commutativity and is defined as

$$\begin{aligned} C_{\mu}{}^{\nu} &\equiv A_{\mu}{}^{\nu} + B_{\mu}{}^{\nu} \\ &\equiv B_{\mu}{}^{\nu} + A_{\mu}{}^{\nu} \end{aligned} \tag{30}$$

2.5.2. Anti tensor I

Definition 2.6 (Anti tensor I).

Let $a_{\mu\nu}$ denote a co-variant (lower index) second-rank tensor. Let $b_{\mu\nu}$, $c_{\mu\nu}$ et cetera denote other co-variant second-rank tensors. Let $E_{\mu\nu}$ denote the sum of these co-variant second-rank tensors. Let the relationship $a_{\mu\nu} + b_{\mu\nu} + c_{\mu\nu} + ... \equiv E_{\mu\nu}$ be given. A co-variant second-rank anti tensor (see also Barukčić, 2020c) of a tensor $a_{\mu\nu}$ denoted in general as $\underline{a}_{\mu\nu}$ is defined

$$\begin{aligned} \underline{a}_{\mu\nu} &\equiv E_{\mu\nu} - a_{\mu\nu} \\ &\equiv b_{\mu\nu} + c_{\mu\nu} + \ldots \end{aligned} \tag{31}$$

2.5.3. Anti tensor II

Definition 2.7 (Anti tensor II).

Let $a^{\mu\nu}$ denote a contra-variant (upper index) second-rank tensor. Let $b^{\mu\nu}$, $c^{\mu\nu}$ et cetera denote other contra-variant (upper index) second-rank tensors. Let $E^{\mu\nu}$ denote the sum of these contra-variant (upper index) second-rank tensors. Let the relationship $a^{\mu\nu} + b^{\mu\nu} + c^{\mu\nu} + ... \equiv E^{\mu\nu}$ be given. A co-variant second-rank anti tensor of a tensor $a^{\mu\nu}$ denoted in general as $\underline{a}^{\mu\nu}$ is defined

$$\begin{aligned} \underline{a}^{\mu\nu} &\equiv E^{\mu\nu} - a^{\mu\nu} \\ &\equiv b^{\mu\nu} + c^{\mu\nu} + \ldots \end{aligned} \tag{32}$$

2.5.4. Anti tensor III

Definition 2.8 (**Anti tensor III**).

Let $a_{\mu}{}^{\nu}$ denote a mixed second-rank tensor. Let $b_{\mu}{}^{\nu}$, $c_{\mu}{}^{\nu}$ et cetera denote other mixed second-rank tensors. Let $E_{\mu}{}^{\nu}$ denote the sum of these mixed second-rank tensors. Let the relationship $a_{\mu}{}^{\nu} + b_{\mu}{}^{\nu} + c_{\mu}{}^{\nu} + ... \equiv E_{\mu}{}^{\nu}$ be given. A mixed second-rank anti tensor of a tensor $a_{\mu}{}^{\nu}$ denoted in general as $\underline{a}_{\mu}{}^{\nu}$ is defined

$$\begin{aligned} \underline{a}_{\mu}{}^{\nu} &\equiv E_{\mu}{}^{\nu} - a_{\mu}{}^{\nu} \\ &\equiv b_{\mu}{}^{\nu} + c_{\mu}{}^{\nu} + \ldots \end{aligned} \tag{33}$$

2.5.5. Tensor subtraction

Definition 2.9 (**Tensor subtraction**).

The subtraction of two second rank co-variant tensors is defined as

$$C_{\mu\nu} \equiv A_{\mu\nu} - B_{\mu\nu} \tag{34}$$

The subtraction of two second rank contra-variant tensors is defined as

$$C^{\mu\nu} \equiv A^{\mu\nu} - B^{\mu\nu} \tag{35}$$

The subtraction of two second rank mixed tensors is defined as

$$C_{\mu}{}^{\nu} \equiv A_{\mu}{}^{\nu} - B_{\mu}{}^{\nu} \tag{36}$$

2.5.6. Symmetric and anti symmetric tensors

Definition 2.10 (Symmetric and anti symmetric tensors).

Symmetric tensors of rank 2 may represent many physical properties of objective reality. A co-variant second-rank tensor $a_{\mu\nu}$ is symmetric if

$$a_{\mu\nu} \equiv a_{\nu\mu} \tag{37}$$

However, there are circumstances, where a tensor is anti-symmetric. A co-variant second-rank tensor $a_{\mu\nu}$ is anti-symmetric if

$$a_{\mu\nu} \equiv -a_{\nu\mu} \tag{38}$$

Thus far, there are circumstances were an anti-tensor is identical with an anti-symmetrical tensor.

$$a_{\mu\nu} \equiv E_{\mu\nu} - b_{\mu\nu} + ... \equiv E_{\mu\nu} - \underline{a}_{\mu\nu} \equiv -a_{\nu\mu} \tag{39}$$

Under conditions where $E_{\mu\nu} = 0$, an anti-tensor is identical with an anti-symmetrical tensor or it is

$$-\underline{a}_{\mu\nu} \equiv -a_{\nu\mu} \tag{40}$$

However, an anti-tensor is not identical with an anti-symmetrical tensor as such.

Definition 2.11 (Multiplication of tensors). *Let g_{kl} or $g_{\mu\nu}$ denote a 2-index metric tensors. Let $g_{kl\mu\nu}$ denote a 4-index metric tensors. Let $g_{kl\mu\nu...}$ denote a n-th index metric tensor. The n-index metric tensor $g_{kl\mu\nu...}$ itself is a covariant symmetric tensor and equally an example of a tensor field. If we pause for a moment today and rely on Einstein's "Die Grundlage der allgemeinen Relativitätstheorie " (see Einstein, 1916, p. 784), it is*

$$g_{kl\mu\nu} \equiv g_{kl} g_{\mu\nu} \tag{41}$$

and in the case of n-th rank order

$$g_{kl\mu\nu...} \equiv g_{kl} g_{\mu\nu...} \tag{42}$$

The mixed and contra-variant cases are similar. Riemann defined the distance between two neighbouring points more or less by a quadratic differential form. The geometry based on the positive definite Riemannian metric tensor is called the Riemannian geometry. However, tensor calculus as a generalization of classical linear algebra should assure that formulae are invariant under coordinate transformations and that the same are independent of any kind of the rank order of the metric tensor chosen. Some rules of the multiplication of tensors are provided to us by Einstein (see Einstein, 1916) himself.

$$T_{\mathrm{a\,b\,c}} \equiv A_{\mathrm{a\,b}} B_{\mathrm{c}} \tag{43}$$

(see Einstein, 1916, p. 784)

Furthermore, it is

$$T^{\mathrm{a\,b\,c\,d}} \equiv A^{\mathrm{a\,b}} B^{\mathrm{c\,d}} \tag{44}$$

(see Einstein, 1916, p. 784)

and equally

$$T^{\mathrm{a\,b}}_{\mathrm{c\,d}} \equiv A^{\mathrm{a\,b}} B_{\mathrm{c\,d}} \tag{45}$$

(see Einstein, 1916, p. 784)

A scalar F, or a tensor of zero rank, is given by the relationship

$$F \equiv F^{\mathrm{b}}_{\mathrm{b}} \equiv F^{\mathrm{a\,b}}_{\mathrm{a\,b}} \equiv F_{\mathrm{a\,b}} F^{\mathrm{a\,b}} \tag{46}$$

(see Einstein, 1916, p. 785)

The covariant and contravariant products of two rank 2 tensors give the same value and result in a scalar. In general, scalar products are operations on two tensors of the same rank that yield a scalar. The relationship (see equation 46, p. 15) is of importance for the fundamental invariants of the electromagnetic field too. A covariant tensor of the second rank type is defined as

$$T_{\mathrm{c\,d}} \equiv A_{\mathrm{c}} B_{\mathrm{d}} \tag{47}$$

(see Einstein, 1916, p. 782)

A contravariant tensor of the second rank type is defined as

$$T^{\mathrm{c\,d}} \equiv A^{\mathrm{c}} B^{\mathrm{d}} \tag{48}$$

(see Einstein, 1916, p. 782)

A mixed tensor of the second rank type is defined by Einstein as follows.

$$T_{\mathrm{c}}^{\mathrm{d}} \equiv A_{\mathrm{c}} B^{\mathrm{d}} \tag{49}$$

(see Einstein, 1916, p. 783)

2.5.7. The metric tensor $g_{\mu\nu}$ and the inverse metric tensor $g^{\mu\nu}$

General relativity is a theory of the geometrical properties of space-time too, while the metric tensor $g_{\mu\nu}$ itself is of fundamental importance for general relativity. The metric tensor $g_{\mu\nu}$ is something like the generalization of the Pythagorean theorem. Thus far, it does not appear to be necessary to restrict the validity of the Pythagorean theorem only to certain situations. The question is justified why the Riemannian geometry should be oppressed by the quadratic restriction. In this context, **Finsler geometry**, named after Paul Finsler (1894 - 1970) who studied it in his doctoral thesis (see Finsler, 1918) in 1918, appears to be a kind of metric generalization of Riemannian geometry without the quadratic restriction and justifies the attempt to systematize and to extend the possibilities of general relativity.

Definition 2.12 (The metric tensor $g_{\mu\nu}$ and the inverse metric tensor $g^{\mu\nu}$). *The distance between any two points in a given space can be described geometrically by a generalized Pythagorean theorem, the metric tensor $g_{\mu\nu}$. Sharing Einstein's point of view, it is in general*

$$g_{\mu\nu} \times g^{\mu\nu} \equiv \delta_{\nu}{}^{\nu} \equiv D \tag{50}$$

where D might denote the number of space-time dimensions. Vectors and scalars are invariant under coordinate transformations. In point of fact, Einstein field equations (Einstein, 1915, 1916, 1917, 1935, Einstein and de Sitter, 1932) were initially formulated by Einstein himself in the context of a four-dimensional theory even though Einstein field equations need not to break down under conditions of D space-time dimensions (see Stephani, 2003). Nonetheless, based on Einstein's statement (Einstein, 1916, p. 796), one gets (see also Einstein, 1923b, p. 91)

$$g_{\mu\nu} \times g^{\mu\nu} \equiv \delta_{\nu}{}^{\nu} \equiv D \equiv +4 \tag{51}$$

or

$$\frac{1}{g_{\mu\nu} \times g^{\mu\nu}} \equiv \frac{1}{4} \tag{52}$$

where $g^{\mu\nu}$ is the matrix inverse of the metric tensor $g_{\mu\nu}$. The inverse metric tensor or the metric tensor, which is always symmetric, allow tensors to be transformed into each other and are used to lower and raise indices. Einstein's point of view is that

> "... in the general theory of relativity ... must be ... the tensor $g_{\mu\nu}$ of the gravitational potential"
> (Einstein, 1923b, p. 88)

Definition 2.13 (The metric tensor $g_{\mu\nu}$ decomposed). *The fundamental difference between the metric tensors of the four basic fields of nature, denoted as $a_{\mu\nu}$, $b_{\mu\nu}$, $c_{\mu\nu}$ and $d_{\mu\nu}$, finds its complete expression in equation 53 as*

$${}_{a}g_{\mu\nu} + {}_{b}g_{\mu\nu} + {}_{c}g_{\mu\nu} + {}_{d}g_{\mu\nu} \equiv g_{\mu\nu} \tag{53}$$

where $_{a}g_{\mu\nu}$ is the metric tensor of the ordinary matter, $_{b}g_{\mu\nu}$ is the metric tensor of electromagnetism, $_{c}g_{\mu\nu}$ is the metric tensor of gravitational field, $_{d}g_{\mu\nu}$ is the metric tensor of gravitational waves and $g_{\mu\nu}$ is the metric tensor of Einstein's general theory of relativity. We distinguish here between the four basic field of nature, as follows. Details are illustrated by table 1.

Table 1. The metric field decomposed

		Curvature		
		YES	NO	
Momentum	YES	$({}_{\mathrm{a}}g_{\mu\nu})$	$({}_{\mathrm{b}}g_{\mu\nu})$	$({}_{\mathrm{E}}g_{\mu\nu})$
	NO	$({}_{\mathrm{d}}g_{\mu\nu})$	$({}_{\mathrm{d}}g_{\mu\nu})$	$({}_{\underline{\mathrm{E}}}g_{\mu\nu})$
		$({}_{\mathrm{G}}g_{\mu\nu})$	$({}_{\underline{\mathrm{G}}}g_{\mu\nu})$	$(g_{\mu\nu})$

In this publication, let $a_{\mu\nu}$, $b_{\mu\nu}$, $c_{\mu\nu}$ and $d_{\mu\nu}$ denote the covariant second rank tensors of the four basic fields of nature where $a_{\mu\nu} \equiv a \times g_{\mu\nu}$ is the stress-energy tensor of ordinary matter, $b_{\mu\nu} \equiv b \times g_{\mu\nu}$ is the stress-energy tensor of electro-magnetic field, $c_{\mu\nu} \equiv c \times g_{\mu\nu}$ and $d_{\mu\nu} \equiv d \times g_{\mu\nu}$ et cetera. Multiplying the relationships of 1 by (R/D), where R is the Ricci scalar and D is spacetime dimension, we obtain the Einstein field equations as outlined by table 2.

Table 2. Decomposed metric field and the Einstein field equations

		Curvature		
		YES	NO	
Momentum	YES	$(R/D)\times({}_{\mathrm{a}}g_{\mu\nu})$	$(R/D)\times({}_{\mathrm{b}}g_{\mu\nu})$	$(R/D)\times({}_{\mathrm{E}}g_{\mu\nu})$
	NO	$(R/D)\times({}_{\mathrm{d}}g_{\mu\nu})$	$(R/D)\times({}_{\mathrm{d}}g_{\mu\nu})$	$(R/D)\times({}_{\underline{\mathrm{E}}}g_{\mu\nu})$
		$(R/D)\times({}_{\mathrm{G}}g_{\mu\nu})$	$(R/D)\times({}_{\underline{\mathrm{G}}}g_{\mu\nu})$	$(R/D)\times(g_{\mu\nu})$

Definition 2.14 (**The metric tensor $_{gw}g_{\mu\nu}$ of gravitational waves**). *Let $g_{\mu\nu}$ denote the metric tensor of Einstein's general theory of relativity. Let $_{gw}g_{\mu\nu}$ denote the metric tensor of* ***gravitational waves*** *of Einstein's general theory of relativity. Let $_{\underline{gw}}g_{\mu\nu}$ denote the metric tensor of* ***anti-gravitational waves*** *of Einstein's general theory of relativity. In general, we define*

$$_{E}g_{\mu\nu} \equiv {}_{\underline{gw}}g_{\mu\nu} + {}_{gw}g_{\mu\nu} \tag{54}$$

Definition 2.15 (**The metric tensor $\eta_{\mu\nu}$ of special relativity**). *There is a fundamental difference between Special and General Relativity regarding the metric tensor. Let $\eta_{\mu\nu}$ denote the metric tensor of Einstein's special theory of relativity. In general, depending upon circumstances, it is*

$$\eta_{\mu\nu} = \begin{Bmatrix} -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & +1 \end{Bmatrix} \quad \textit{(see Einstein, 1916, p. 778).}$$

*Let $\underline{\eta}_{\mu\nu}$ denote **the anti-metric tensor of Einstein's special theory of relativity**. Let $g_{\mu\nu}$ denote the metric tensor of Einstein's general theory of relativity. In general, it is (see equation 31)*

$$g_{\mu\nu} \equiv \eta_{\mu\nu} + \underline{\eta}_{\mu\nu} \tag{55}$$

There might exist circumstances where ${}_{d}g_{\mu\nu} \equiv {}_{gw}g_{\mu\nu} \equiv \underline{\eta}_{\mu\nu}$. The n-th index relationship follows (see equation 31) as

$$g_{kl\mu\nu\ldots} \equiv \eta_{kl\mu\nu\ldots} + \underline{\eta}_{kl\mu\nu\ldots} \tag{56}$$

Definition 2.16 (**Kronecker delta**). *The Kronecker delta (see Zehfuss, 1858), a notation invented by Leopold Kronecker (1823-1891) in 1868 (see Kronecker, 1868) appears in many areas of physics, mathematics, and engineering and is defined as*

$$g_{\mu\rho} \times g^{\nu\rho} \equiv g_{\mu}{}^{\nu} \equiv \delta_{\mu}{}^{\nu} \tag{57}$$

Technically, the Kronecker delta (see Einstein, 1916, p. 787) itself is a mixed second-rank tensor. The quantity

$$\delta_{i}{}^{i} \equiv \delta_{1}{}^{1} + \delta_{2}{}^{2} + \ldots + \delta_{D}{}^{D} \equiv D \tag{58}$$

is an invariant.

Definition 2.17 (**Index raising**). *According to Einstein (see also Einstein, 1916, p. 790), it is*

$$F_{\mu\nu} \equiv g_{\mu\alpha} g_{\nu\beta} F^{\alpha\beta} \tag{59}$$

and equally

$$F^{\mu\nu} \equiv g^{\mu\alpha} g^{\nu\beta} F_{\alpha\beta} \tag{60}$$

In other (Kay, 1988) words (see Einstein, 1916, p. 790), an order-2 tensor, twice multiplied by the contra-variant metric tensor and contracted (Einstein, 1916, p. 785) in different indices, raises each index. It is

$$F^{\left(\begin{smallmatrix}1 & 3\\ \mu & c\end{smallmatrix}\right)} \equiv g^{\left(\begin{smallmatrix}1 & 2\\ \mu & \nu\end{smallmatrix}\right)} \times g^{\left(\begin{smallmatrix}3 & 4\\ c & d\end{smallmatrix}\right)} \times F_{\left(\begin{smallmatrix}\nu & d\\ 2 & 4\end{smallmatrix}\right)} \tag{61}$$

or more professionally

$$F^{\mu\ c} \equiv g^{\mu\nu} \times g^{cd} \times F_{\nu\ d} \tag{62}$$

Furthermore, it is (see Einstein, 1916, p. 790)

$$A^{\mu\nu} \equiv g^{\mu a} g^{\nu b} A_{ab} \tag{63}$$

or

$$A_{\mu\nu} \equiv g_{\mu a} g_{\nu b} A^{ab} \tag{64}$$

et cetera. Following Einstein, it is $g_{\mu\nu} \times g^{\mu\nu} \equiv \delta_{\mu}{}^{\mu}$ (Einstein, 1916, p. 796). Furthermore, in conjunction with another view (see equation 46, p. 15) of Einstein (see Einstein, 1916, p. 785), it is

$$F \equiv F_{\mu\nu}{}^{\mu\nu} \equiv F_{\mu\nu} \times F^{\mu\nu} \tag{65}$$

2.6. *Extended tensor algebra*

In the following, for the sake of better understanding, we consider tensors of order two. As is known, the components of a tensor of order two can be displayed in 4 × 4 matrix form.

2.6.1. Zero tensor

Definition 2.18 (Zero tensor).

The second-rank co-variant zero tensor is defined as

$$0_{\mu\nu} \equiv \underbrace{\begin{pmatrix} 0_{00} & 0_{01} & 0_{02} & 0_{03} \\ 0_{10} & 0_{11} & 0_{12} & 0_{13} \\ 0_{20} & 0_{21} & 0_{22} & 0_{23} \\ 0_{30} & 0_{31} & 0_{32} & 0_{33} \end{pmatrix}}_{0_{\mu\nu}\ tensor} \tag{66}$$

This definition is also valid for contra-variant or mixed tensors too.

2.6.2. The negation of one

Definition 2.19 (The negation of one).

The negation of one, denoted as $\neg(1)$, is defined by division as

$$\neg(1) = \frac{0}{1} \tag{67}$$

In general, it is

$$\neg(1) \times 1 = +1 - 1 = \frac{0}{1} \times 1 = \frac{1}{1} \times 0 = 0 \tag{68}$$

The negation of one, denoted as $\neg$, is defined by subtraction as

$$\neg = 1 - \tag{69}$$

In general, it is

$$\neg 1 = 1 - 1 = 0 \tag{70}$$

2.6.3. Unity tensor

Definition 2.20 (Unity tensor).

The second-rank co-variant unity tensor is defined as

$$1_{\mu\nu} \equiv \underbrace{\begin{pmatrix} 1_{00} & 1_{01} & 1_{02} & 1_{03} \\ 1_{10} & 1_{11} & 1_{12} & 1_{13} \\ 1_{20} & 1_{21} & 1_{22} & 1_{23} \\ 1_{30} & 1_{31} & 1_{32} & 1_{33} \end{pmatrix}}_{1_{\mu\nu}\ tensor} \tag{71}$$

This definition is also valid for contra-variant or mixed tensors too.

2.6.4. The negation of zero

Definition 2.21 (The negation of zero).

The negation of zero, denoted as $\neg(0)$, is defined by division as

$$\neg(0) = \underline{0} = \frac{1}{0} \tag{72}$$

In general, it is

$$\neg(0) \times 0 = \underline{0} \times 0 = \frac{1}{0} \times 0 = \frac{0}{0} = 1 \tag{73}$$

The negation of zero, denoted as $\neg(0)$ or as $\underline{0}$, is defined by subtraction as

$$\neg = 1 - \tag{74}$$

In general, it is

$$\neg 0 = \underline{0} = 1 - 0 = 1 \tag{75}$$

2.6.5. The tensor of the number 2

Definition 2.22 (The tensor of the number 2).

The second-rank co-variant tensor of the number 2 is defined as

$$2_{\mu\nu} \equiv \underbrace{\begin{pmatrix} 2_{00} & 2_{01} & 2_{02} & 2_{03} \\ 2_{10} & 2_{11} & 2_{12} & 2_{13} \\ 2_{20} & 2_{21} & 2_{22} & 2_{23} \\ 2_{30} & 2_{31} & 2_{32} & 2_{33} \end{pmatrix}}_{2_{\mu\nu}\ tensor} \tag{76}$$

This definition is also valid for contra-variant or mixed tensors an other numbers too. Whether it makes sense to define numbers or scalars et cetera in the form of a tensor is worth being discussed. However, such an approach has various advantages too.

2.6.6. Speed of the light tensor

Definition 2.23 (Speed of the light tensor).

Scientists and thinkers have been fascinated by the speed of light since ever. Aristotle (384-322 BCE) himself has been of the opinion that the speed of light was infinite. Let c denote the speed of the light in vacuum. The second-rank co-variant tensor of speed of the light is defined as

$$c_{\mu\nu} \equiv \underbrace{\begin{pmatrix} c_{00} & c_{01} & c_{02} & c_{03} \\ c_{10} & c_{11} & c_{12} & c_{13} \\ c_{20} & c_{21} & c_{22} & c_{23} \\ c_{30} & c_{31} & c_{32} & c_{33} \end{pmatrix}}_{c_{\mu\nu}\ tensor} \tag{77}$$

2.6.7. Archimedes' constant tensor

Definition 2.24 (Archimedes' constant tensor).

The second-rank co-variant tensor of the Archimedes of Syracuse (c. 287 – c. 212 B. C. E.) constant π is defined as

$$\pi_{\mu\nu} \equiv \underbrace{\begin{pmatrix} \pi_{00} & \pi_{01} & \pi_{02} & \pi_{03} \\ \pi_{10} & \pi_{11} & \pi_{12} & \pi_{13} \\ \pi_{20} & \pi_{21} & \pi_{22} & \pi_{23} \\ \pi_{30} & \pi_{31} & \pi_{32} & \pi_{33} \end{pmatrix}}_{\pi_{\mu\nu}\ tensor} \tag{78}$$

This definition is also valid for contra-variant or mixed tensors too.

2.6.8. Newton's constant tensor

Definition 2.25 (Newton's constant tensor).

The second-rank co-variant tensor of the Newton's constant (see Newton, 1687, p. 198) is defined, as

$$\gamma_{\mu\nu} \equiv \underbrace{\begin{pmatrix} \gamma_{00} & \gamma_{01} & \gamma_{02} & \gamma_{03} \\ \gamma_{10} & \gamma_{11} & \gamma_{12} & \gamma_{13} \\ \gamma_{20} & \gamma_{21} & \gamma_{22} & \gamma_{23} \\ \gamma_{30} & \gamma_{31} & \gamma_{32} & \gamma_{33} \end{pmatrix}}_{\gamma_{\mu\nu}\ tensor} \tag{79}$$

This definition is also valid for contra-variant or mixed tensors too.

2.6.9. Planck's constant tensor

Definition 2.26 (Planck's constant tensor).

Max Karl Ernst Ludwig Planck (1858-1947) quantized the energy ${}_{\mathrm{R}}\mathrm{E}_{\mathrm{t}}$ as

$${}_{\mathrm{R}}E_{\mathrm{t}} \equiv n \times h \times {}_{\mathrm{R}}f_{\mathrm{t}} \tag{80}$$

where h is Planck's constant (Planck, 1901), ${}_{\mathrm{R}}\mathrm{f}_{\mathrm{t}}$ is the frequency and n is an integer number. In the following, Paul Adrien Maurice Dirac (1902-1984) defined the so-called Dirac's constant $\hbar$ (Dirac, 1926) as

$$\begin{aligned} h &\equiv 2 \times \pi \times \hbar \\ &\equiv \pi \times (2 \times \hbar) \\ &\equiv \pi \times s \end{aligned} \tag{81}$$

Plato (424/423 – 348/347 BCE), a Greek philosopher born in Athens, defined a circle as follows

> "Rund ist doch das, dessen Enden überall gleich weit von der Mitte entfernt sind? "
>
> (see also Plato, 1910, p. 26)

Figure 3 might illustrate these basic relationships.

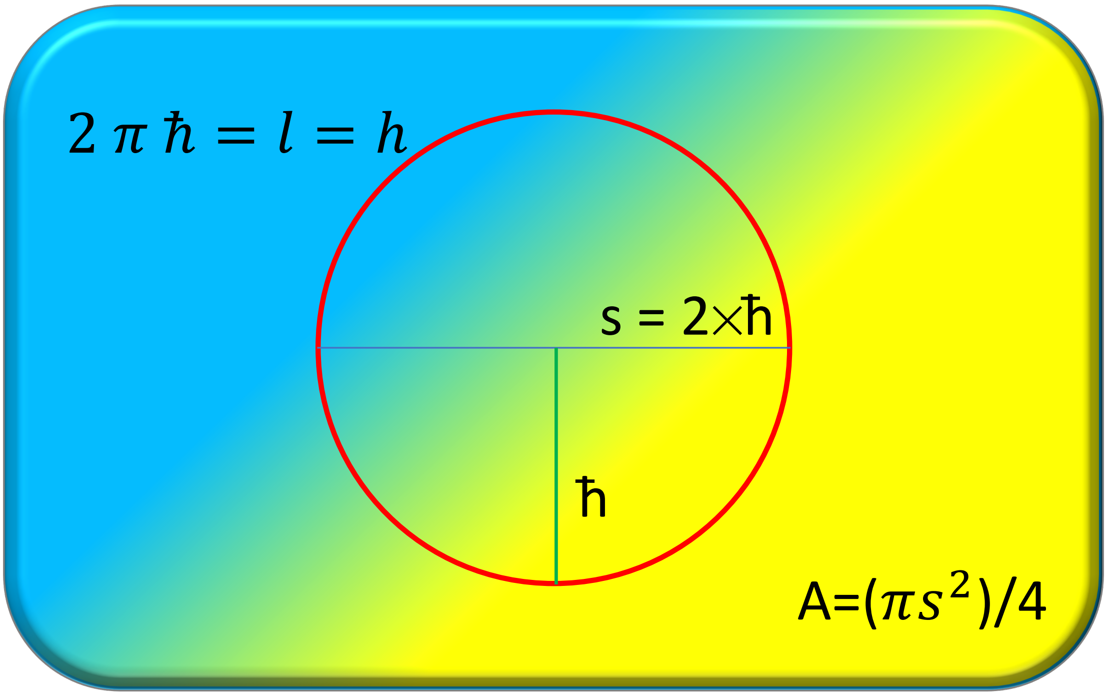

Figure 3. Planck's constant h, quantum loop and string theory.

A few thoughts - which are necessarily first thoughts - might consider circumstances where h can be regarded as a loop, denoted as l, of the background-independent quantization of general relativity by quantum loop (see Ashtekar and Bianchi, 2021, Ashtekar and Geroch, 1974, Rovelli, 2008) theory, while s is treated as a string of string (see Bergshoeff et al., 1987, Green and Schwarz, 1982) theory. However, "Strings and loop gravity may not necessarily be competing theories: there might be a sort of complementarity, at least methodological, between the two. Indeed, the open problems of string theory mostly concern its background-independent formulation, while loop quantum gravity is precisely a set of techniques for dealing with background-independent theories. Perhaps the two approaches might even, to some extent, converge. "[5] Under these conditions, it is

$$l \equiv \pi \times s \tag{82}$$

or

$$\pi \equiv \frac{l}{s} \tag{83}$$

Equation 83 implies due to our experience that π can hardly be treated as a constant. In this context,

[5]Rovelli C. Loop Quantum Gravity. Living Rev Relativ. 2008;11(1):5. doi: 10.12942/lrr-2008-5. Epub 2008 Jul 15. PMID: 28179822; PMCID: PMC5256093.

the second-rank co-variant tensor of Planck's constant h (Planck, 1901) is defined, as

$$h_{\mu\nu} \equiv \underbrace{\begin{pmatrix} h_{00} & h_{01} & h_{02} & h_{03} \\ h_{10} & h_{11} & h_{12} & h_{13} \\ h_{20} & h_{21} & h_{22} & h_{23} \\ h_{30} & h_{31} & h_{32} & h_{33} \end{pmatrix}}_{h_{\mu\nu}\ tensor} \tag{84}$$

This definition is also valid for contra-variant or mixed tensors too.

2.6.10. Dirac's constant tensor

Definition 2.27 (Dirac's constant tensor).

The second-rank co-variant tensor of Dirac's constant $\hbar$ (Dirac, 1926) is defined, as

$$\hbar_{\mu\nu} \equiv \underbrace{\begin{pmatrix} \hbar_{00} & \hbar_{01} & \hbar_{02} & \hbar_{03} \\ \hbar_{10} & \hbar_{11} & \hbar_{12} & \hbar_{13} \\ \hbar_{20} & \hbar_{21} & \hbar_{22} & \hbar_{23} \\ \hbar_{30} & \hbar_{31} & \hbar_{32} & \hbar_{33} \end{pmatrix}}_{\hbar_{\mu\nu}\ tensor} \tag{85}$$

This definition is also valid for contra-variant or mixed tensors too.

2.6.11. The commutative multiplication of tensors

Definition 2.28 (The commutative multiplication of tensors).

Multiplication is something which is equivalent to a repeated addition. Addition itself has the properties of associativity and commutativity. The question is justified whether there might exist something like a commutative multiplication of tensors. Let $U_{\mu\nu}$ denote a second-rank tensor. Let $W_{\mu\nu}$ denote another second-rank tensor. The commutative multiplication of two second-rank tensors is defined as an entry wise multiplication of both tensors. It is,

$$U_{\mu\nu} \cap W_{\mu\nu} \equiv X_{\mu\nu} \tag{86}$$

where the sign $\cap$ denotes a commutative multiplication of tensors of the same rank. The commutative multiplication of two tensors of the same rank is commutative, associative and distributive.

Example.

Example of an entrywise multiplication of two tensors of the same rank.

$$\underbrace{\begin{pmatrix} u_{00} & u_{01} & u_{02} & u_{03} \\ u_{10} & u_{11} & u_{12} & u_{13} \\ u_{20} & u_{21} & u_{22} & u_{23} \\ u_{30} & u_{31} & u_{32} & u_{33} \end{pmatrix}}_{U_{\mu\nu}} \cap \underbrace{\begin{pmatrix} w_{00} & w_{01} & w_{02} & w_{03} \\ w_{10} & w_{11} & w_{12} & w_{13} \\ w_{20} & w_{21} & w_{22} & w_{23} \\ w_{30} & w_{31} & w_{32} & w_{33} \end{pmatrix}}_{W_{\mu\nu}}$$
$$= \underbrace{\begin{pmatrix} (u_{00} \times w_{00}) & (u_{01} \times w_{01}) & (u_{02} \times w_{02}) & (u_{03} \times w_{03}) \\ (u_{10} \times w_{10}) & (u_{11} \times w_{11}) & (u_{12} \times w_{12}) & (u_{13} \times w_{13}) \\ (u_{20} \times w_{20}) & (u_{21} \times w_{21}) & (u_{22} \times w_{22}) & (u_{23} \times w_{23}) \\ (u_{30} \times w_{30}) & (u_{31} \times w_{31}) & (u_{32} \times w_{32}) & (u_{33} \times w_{33}) \end{pmatrix}}_{X_{\mu\nu}} \tag{87}$$

Jacques Salomon Hadamard (1865-1963), a French mathematician, defined a similar operation of two matrices of the same dimension $i \times j$ (see also Hadamard, 1893) which is commutative, associative and distributive. The Hadamard product (also known as the Issai Schur (see also Schur, 1911, p. 14) (1875 – 1941) product (see also Davis, 1962) or the point wise product is of use for a commutative matrix multiplication and is defined something as

$$(u \circ w)_{\mathrm{ij}} \equiv u_{\mathrm{ij}} w_{\mathrm{ij}} \tag{88}$$

where the sign $\circ$ denotes an entry wise matrix multiplication.

2.6.12. The tensor double dot product on the closest indices

Definition 2.29 (**The tensor double dot product on the closest indices**).

Two tensors can be contracted over the first two indices of the second tensor or over the last two indices of the first tensor (double contraction). As is known, a double dot product between two tensors of orders m and n will result in a tensor of order (m + n - 4). Let $u_{\mu\nu}$ and $w_{\mu\nu}$ denote two second-rank tensors. Let : denote the contraction of two tensors $u_{\mu\nu}$ and $w_{\mu\nu}$ on the closest indices, then

$$u : w = u_{\mu\nu} w_{\nu\mu} \tag{89}$$

2.6.13. The tensor double dot product on the non-closest indices

Definition 2.30 (**The tensor double dot product on the non-closest indices**).

Let $u_{\mu\nu}$ and $w_{\mu\nu}$ denote two second-rank tensors. Let $\underline{:}$ denote the contraction of two tensors $u_{\mu\nu}$ and $w_{\mu\nu}$ on the non-closest indices, then

$$u \underline{:} w = u_{\mu\nu} w_{\mu\nu} \tag{90}$$

Especially under conditions where both second-rank tensors are symmetric, both definitions of the tensor double dot product coincide but not necessarily in general.

2.6.14. The division of tensors

Definition 2.31 (The division of tensors).

Division is something which is related to multiplication. Let $a_{\mu\nu}$ denote a second-rank tensor. Let $b_{\mu\nu}$ denote another second-rank tensor. Let $U_{\mu\nu}$ denote another second-rank co-variant tensor. In general, let it be that

$$a_{\mu\nu} + b_{\mu\nu} \equiv U_{\mu\nu} \tag{91}$$

The probability of a tensor $a_{\mu\nu}$, denoted as $p(a_{\mu\nu})$, is calculated entry wise as follows.

$$p(a_{\mu\nu}) \equiv \begin{pmatrix} a_{00} & a_{01} & a_{02} & a_{03} \\ a_{10} & a_{11} & a_{12} & a_{13} \\ a_{20} & a_{21} & a_{22} & a_{23} \\ a_{30} & a_{31} & a_{32} & a_{33} \end{pmatrix} / \begin{pmatrix} U_{00} & U_{01} & U_{02} & U_{03} \\ U_{10} & U_{11} & U_{12} & U_{13} \\ U_{20} & U_{21} & U_{22} & U_{23} \\ U_{30} & U_{31} & U_{32} & U_{33} \end{pmatrix} \equiv \begin{pmatrix} \frac{a_{00}}{U_{00}} & \frac{a_{01}}{U_{01}} & \frac{a_{02}}{U_{02}} & \frac{a_{03}}{U_{03}} \\ \frac{a_{10}}{U_{10}} & \frac{a_{11}}{U_{11}} & \frac{a_{12}}{U_{12}} & \frac{a_{13}}{U_{13}} \\ \frac{a_{20}}{U_{20}} & \frac{a_{21}}{U_{21}} & \frac{a_{22}}{U_{22}} & \frac{a_{23}}{U_{23}} \\ \frac{a_{30}}{U_{30}} & \frac{a_{31}}{U_{31}} & \frac{a_{32}}{U_{32}} & \frac{a_{33}}{U_{33}} \end{pmatrix} \tag{92}$$

2.6.15. The exponentiation of a tensor to the power n

Definition 2.32 (The exponentiation of a tensor to the power n)**.**

A second-rank co-variant tensor to the power n, denoted by ${}^{n}U_{\mu\nu}$, is determined by the fact that every single component of such a tensor is multiplied by itself n-times. In general, it is

$$\begin{aligned} {}^{n}U_{\mu\nu} &= \underbrace{\begin{pmatrix} \underbrace{(u_{00} \times u_{00} \times \ldots)}_{n-times} & \underbrace{(u_{01} \times u_{01} \times \ldots)}_{n-times} & \underbrace{(u_{02} \times u_{02} \times \ldots)}_{n-times} & \underbrace{(u_{03} \times u_{03} \times \ldots)}_{n-times} \\ \underbrace{(u_{10} \times u_{10} \times \ldots)}_{n-times} & \underbrace{(u_{11} \times u_{11} \times \ldots)}_{n-times} & \underbrace{(u_{12} \times u_{12} \times \ldots)}_{n-times} & \underbrace{(u_{13} \times u_{13} \times \ldots)}_{n-times} \\ \underbrace{(u_{20} \times u_{20} \times \ldots)}_{n-times} & \underbrace{(u_{21} \times u_{21} \times \ldots)}_{n-times} & \underbrace{(u_{22} \times u_{22} \times \ldots)}_{n-times} & \underbrace{(u_{23} \times u_{23} \times \ldots)}_{n-times} \\ \underbrace{(u_{30} \times u_{30} \times \ldots)}_{n-times} & \underbrace{(u_{31} \times u_{31} \times \ldots)}_{n-times} & \underbrace{(u_{32} \times u_{32} \times \ldots)}_{n-times} & \underbrace{(u_{33} \times u_{33} \times \ldots)}_{n-times} \end{pmatrix}}_{{}^{n}U_{\mu\nu}} \\ &= \underbrace{\begin{pmatrix} (u_{00})^{n} & (u_{01})^{n} & (u_{02})^{n} & (u_{03})^{n} \\ (u_{10})^{n} & (u_{11})^{n} & (u_{12})^{n} & (u_{13})^{n} \\ (u_{20})^{n} & (u_{21})^{n} & (u_{22})^{n} & (u_{23})^{n} \\ (u_{30})^{n} & (u_{31})^{n} & (u_{32})^{n} & (u_{33})^{n} \end{pmatrix}}_{{}^{n}U_{\mu\nu}} \end{aligned} \tag{93}$$

This definition is also valid for contra-variant or mixed tensors too.

2.6.16. The exponentiation of a tensor to the power 1/n

Definition 2.33 (The exponentiation of a tensor to the power 1/n)**.**

A second-rank co-variant tensor to the power n, denoted by ${}^{n}U_{\mu\nu}$, is determined by the fact that every single component of such a tensor is multiplied by itself (1/n)-times. In general, it is

$$
{}^{1/n}U_{\mu\nu} = \underbrace{\begin{pmatrix} (u_{00})^{1/n} & (u_{01})^{1/n} & (u_{02})^{1/n} & (u_{03})^{1/n} \\ (u_{10})^{1/n} & (u_{11})^{1/n} & (u_{12})^{1/n} & (u_{13})^{1/n} \\ (u_{20})^{1/n} & (u_{21})^{1/n} & (u_{22})^{1/n} & (u_{23})^{1/n} \\ (u_{30})^{1/n} & (u_{31})^{1/n} & (u_{32})^{1/n} & (u_{33})^{1/n} \end{pmatrix}}_{{}^{1/n}U_{\mu\nu}} \tag{94}
$$

This definition is also valid for contra-variant or mixed tensors too.

2.6.17. The expectation value of a co-variant second rank tensor

Let $E({}_{R}U_{\mu\nu})$ denote the expectation value of a co-variant second rank tensor ${}_{R}U_{\mu\nu}$. Let $p({}_{R}U_{\mu\nu})$ denote the probability of a tensor ${}_{R}U_{\mu\nu}$. In general, we define

$$
E\left({}_{R}U_{\mu\nu}\right) \equiv p\left({}_{R}U_{\mu\nu}\right) \cap {}_{R}U_{\mu\nu} \tag{95}
$$

and equally

$$
{}^{2}E\left({}_{R}U_{\mu\nu}\right) \equiv E\left({}_{R}U_{\mu\nu}\right) \cap E\left({}_{R}U_{\mu\nu}\right) \equiv p\left({}_{R}U_{\mu\nu}\right) \cap p\left({}_{R}U_{\mu\nu}\right) \cap {}_{R}U_{\mu\nu} \cap {}_{R}U_{\mu\nu} \tag{96}
$$

Let $E({}_{R}U_{kl\mu\nu\ldots})$ denote the expectation value of a co-variant n-index rank tensor ${}_{R}U_{kl\mu\nu\ldots}$. Let $p({}_{R}U_{kl\mu\nu\ldots})$ denote the probability of a co-variant n-index rank tensor ${}_{R}U_{kl\mu\nu\ldots}$. In general, we define expectation value of a co-variant n-index rank tensor

$$
E\left({}_{R}U_{kl\mu\nu\ldots}\right) \equiv p\left({}_{R}U_{kl\mu\nu\ldots}\right) \cap {}_{R}U_{kl\mu\nu\ldots} \tag{97}
$$

It is equally true that

$$
{}^{2}E\left({}_{R}U_{kl\mu\nu\ldots}\right) \equiv E\left({}_{R}U_{kl\mu\nu\ldots}\right) \cap E\left({}_{R}U_{kl\mu\nu\ldots}\right) \equiv p\left({}_{R}U_{kl\mu\nu\ldots}\right) \cap p\left({}_{R}U_{kl\mu\nu\ldots}\right) \cap {}_{R}U_{kl\mu\nu\ldots} \cap {}_{R}U_{kl\mu\nu\ldots} \tag{98}
$$

2.6.18. The expectation value of a second rank anti tensor

Let E($_{\mathrm{R}}\underline{U}_{\mu\nu}$) denote the expectation value of the covariant second rank anti tensor $_{\mathrm{R}}\underline{U}_{\mu\nu}$. Let p($_{\mathrm{R}}\underline{U}_{\mu\nu}$) denote the probability of an anti tensor $_{\mathrm{R}}\underline{U}_{\mu\nu}$. In general, we define

$$\begin{aligned} E\left({}_{\mathrm{R}}\underline{U}_{\mu\nu}\right) &\equiv p\left({}_{\mathrm{R}}\underline{U}_{\mu\nu}\right) \cap U_{\mu\nu} \\ &\equiv \left(1_{\mu\nu} - p\left({}_{\mathrm{R}}U_{\mu\nu}\right)\right) \cap {}_{\mathrm{R}}U_{\mu\nu} \end{aligned} \tag{99}$$

Euclid's theorem is a fundamental statement of geometry and has been proved by Euclid in his famous work Elements. According to Euclid's theorem, it is

$${}_{\mathrm{R}}U_{\mu\nu} \equiv E\left({}_{\mathrm{R}}U_{\mu\nu}\right) + E\left({}_{\mathrm{R}}\underline{U}_{\mu\nu}\right) \tag{100}$$

Theorem 2.1. *It is*

$${}_{R}U_{\mu\nu} \equiv E\left({}_{R}U_{\mu\nu}\right) + E\left({}_{R}\underline{U}_{\mu\nu}\right) \tag{101}$$

Proof. According to Euclid's theorem, it is

$${}_{\mathrm{R}}U_{\mathrm{t}} \equiv E\left({}_{\mathrm{R}}U_{\mathrm{t}}\right) + E\left({}_{\mathrm{R}}\underline{U}_{\mathrm{t}}\right) \tag{102}$$

Multiply $_{\mathrm{R}}U_{\mathrm{t}}$ by the metric tensor $g_{\mu\nu}$ or just define

$${}_{\mathrm{R}}U_{\mathrm{t}} = {}_{\mathrm{R}}U_{\mu\nu} \tag{103}$$

Then the conclusion is true that

$${}_{\mathrm{R}}U_{\mu\nu} \equiv E\left({}_{\mathrm{R}}U_{\mu\nu}\right) + E\left({}_{\mathrm{R}}\underline{U}_{\mu\nu}\right) \tag{104}$$

□

♡

2.6.19. The expectation value of a second rank tensor raised to rower 2

Let E($^{2}{}_{\mathrm{R}}U_{\mu\nu}$) denote the expectation value of the covariant second rank tensor $_{\mathrm{R}}U_{\mu\nu}$ raised to the power 2. Let p($_{\mathrm{R}}U_{\mu\nu}$) denote the probability of a tensor $_{\mathrm{R}}U_{\mu\nu}$. In general, we define

$$\begin{aligned} E\left({}^{2}{}_{\mathrm{R}}U_{\mu\nu}\right) &\equiv p\left({}_{\mathrm{R}}U_{\mu\nu}\right) \cap {}_{\mathrm{R}}U_{\mu\nu} \cap {}_{\mathrm{R}}U_{\mu\nu} \\ &\equiv p\left({}_{\mathrm{R}}U_{\mu\nu}\right) \cap \left({}^{2}{}_{\mathrm{R}}U_{\mu\nu}\right) \end{aligned} \tag{105}$$

Let E($^{2}{}_{\mathrm{R}}U_{\mathrm{kl}\mu\nu\ldots}$) denote the expectation value of a co-variant n-index rank tensor $^{2}{}_{\mathrm{R}}U_{\mathrm{kl}\mu\nu\ldots}$ raised to rower 2. Let p($_{\mathrm{R}}U_{\mathrm{kl}\mu\nu\ldots}$) denote the probability of a co-variant n-index rank tensor $_{\mathrm{R}}U_{\mathrm{kl}\mu\nu\ldots}$. In general, we define the expectation value of a co-variant n-index rank tensor raised to rower 2 as

$$E\left({}^{2}{}_{\mathrm{R}}U_{\mathrm{kl}\mu\nu\ldots}\right) \equiv p\left({}_{\mathrm{R}}U_{\mathrm{kl}\mu\nu\ldots}\right) \cap {}_{\mathrm{R}}U_{\mathrm{kl}\mu\nu\ldots} \cap {}_{\mathrm{R}}U_{\mathrm{kl}\mu\nu\ldots} \tag{106}$$

2.6.20. The variance of a tensor

Definition 2.34 (The variance of a tensor).

Let ${}_{\mathrm{R}}\mathrm{U}_{\mu\nu}$ denote a second-rank co-variant tensor. Let $\mathrm{p}({}_{\mathrm{R}}\mathrm{U}_{\mu\nu})$ denote the probability of a tensor ${}_{\mathrm{R}}\mathrm{U}_{\mu\nu}$. The variance of a tensor ${}_{\mathrm{R}}\mathrm{U}_{\mu\nu}$, denoted as ${}^{2}\sigma\left({}_{\mathrm{R}}U_{\mu\nu}\right)$, is defined as

$$\begin{aligned}
{}^{2}\sigma\left({}_{\mathrm{R}}U_{\mu\nu}\right) &\equiv E\left({}^{2}_{\mathrm{R}}U_{\mu\nu}\right) -{}^{2}\left(E\left({}_{\mathrm{R}}U_{\mu\nu}\right)\right)\\
&\equiv \left(p\left({}_{\mathrm{R}}U_{\mu\nu}\right)\cap{}_{\mathrm{R}}U_{\mu\nu}\cap{}_{\mathrm{R}}U_{\mu\nu}\right)-\left(p\left({}_{\mathrm{R}}U_{\mu\nu}\right)\cap{}_{\mathrm{R}}U_{\mu\nu}\cap p\left({}_{\mathrm{R}}U_{\mu\nu}\right)\cap{}_{\mathrm{R}}U_{\mu\nu}\right)\\
&\equiv {}_{\mathrm{R}}U_{\mu\nu}\cap{}_{\mathrm{R}}U_{\mu\nu}\cap p\left({}_{\mathrm{R}}U_{\mu\nu}\right)\cap\left(1_{\mu\nu}-p\left({}_{\mathrm{R}}U_{\mu\nu}\right)\right)
\end{aligned} \tag{107}$$

From equation 107 follows that

$${}_{\mathrm{R}}U_{\mu\nu}\cap{}_{\mathrm{R}}U_{\mu\nu}\equiv\frac{{}^{2}\sigma\left({}_{\mathrm{R}}U_{\mu\nu}\right)}{p\left({}_{\mathrm{R}}U_{\mu\nu}\right)\cap\left(1_{\mu\nu}-p\left({}_{\mathrm{R}}U_{\mu\nu}\right)\right)} \tag{108}$$

and that

$${}_{\mathrm{R}}U_{\mu\nu}\equiv\frac{\sigma\left({}_{\mathrm{R}}U_{\mu\nu}\right)}{{}^{1/2}\left(p\left({}_{\mathrm{R}}U_{\mu\nu}\right)\cap\left(1_{\mu\nu}-p\left({}_{\mathrm{R}}U_{\mu\nu}\right)\right)\right)} \tag{109}$$

The standard deviation of a second-rank tensor, denoted as $\sigma\left({}_{\mathrm{R}}U_{\mu\nu}\right)$, would follow as

$$\begin{aligned}
\sigma\left({}_{\mathrm{R}}U_{\mu\nu}\right) &\equiv{}^{1/2}\left({}_{\mathrm{R}}U_{\mu\nu}\cap{}_{\mathrm{R}}U_{\mu\nu}\cap p\left({}_{\mathrm{R}}U_{\mu\nu}\right)\cap\left(\left(1_{\mu\nu}-p\left({}_{\mathrm{R}}U_{\mu\nu}\right)\right)\right)\right)\\
&\equiv\sqrt[2]{\left({}_{\mathrm{R}}U_{\mu\nu}\cap{}_{\mathrm{R}}U_{\mu\nu}\cap p\left({}_{\mathrm{R}}U_{\mu\nu}\right)\cap\left(\left(1_{\mu\nu}-p\left({}_{\mathrm{R}}U_{\mu\nu}\right)\right)\right)\right)}
\end{aligned} \tag{110}$$

Let ${}_{\mathrm{R}}\mathrm{U}_{\mathrm{kl}\mu\nu\ldots}$ denote a co-variant n-index rank tensor. Let $\mathrm{p}({}_{\mathrm{R}}\mathrm{U}_{\mathrm{klkl}\mu\nu\ldots\ldots})$ denote the probability of a co-variant n-index rank tensor ${}_{\mathrm{R}}\mathrm{U}_{\mathrm{kl}\mu\nu\ldots}$. The variance of a co-variant n-index rank tensor ${}_{\mathrm{R}}\mathrm{U}_{\mathrm{kl}\mu\nu\ldots}$, denoted as ${}^{2}\sigma\left({}_{\mathrm{R}}U_{\mathrm{kl}\mu\nu\ldots}\right)$, is defined as

$$\begin{aligned}
&{}^{2}\sigma\left({}_{\mathrm{R}}U_{\mathrm{kl}\mu\nu\ldots}\right)\\
&\equiv E\left({}^{2}_{\mathrm{R}}U_{\mathrm{kl}\mu\nu\ldots}\right)-{}^{2}\left(E\left({}_{\mathrm{R}}U_{\mathrm{kl}\mu\nu\ldots}\right)\right)\\
&\equiv\left(p\left({}_{\mathrm{R}}U_{\mathrm{kl}\mu\nu\ldots}\right)\cap{}_{\mathrm{R}}U_{\mathrm{kl}\mu\nu\ldots}\cap{}_{\mathrm{R}}U_{\mathrm{kl}\mu\nu\ldots}\right)-\left(p\left({}_{\mathrm{R}}U_{\mathrm{kl}\mu\nu\ldots}\right)\cap{}_{\mathrm{R}}U_{\mathrm{kl}\mu\nu\ldots}\cap p\left({}_{\mathrm{R}}U_{\mathrm{kl}\mu\nu\ldots}\right)\cap{}_{\mathrm{R}}U_{\mathrm{kl}\mu\nu\ldots}\right)\\
&\equiv{}_{\mathrm{R}}U_{\mathrm{kl}\mu\nu\ldots}\cap{}_{\mathrm{R}}U_{\mathrm{kl}\mu\nu\ldots}\cap p\left({}_{\mathrm{R}}U_{\mathrm{kl}\mu\nu\ldots}\right)\cap\left(1_{\mathrm{kl}\mu\nu\ldots}-p\left({}_{\mathrm{R}}U_{\mathrm{kl}\mu\nu\ldots}\right)\right)
\end{aligned} \tag{111}$$

From equation 111 follows that

$${}_{\mathrm{R}}U_{\mathrm{kl}\mu\nu\ldots}\cap{}_{\mathrm{R}}U_{\mathrm{kl}\mu\nu\ldots}\equiv\frac{{}^{2}\sigma\left({}_{\mathrm{R}}U_{\mathrm{kl}\mu\nu\ldots}\right)}{p\left({}_{\mathrm{R}}U_{\mathrm{kl}\mu\nu\ldots}\right)\cap\left(1_{\mathrm{kl}\mu\nu\ldots}-p\left({}_{\mathrm{R}}U_{\mathrm{kl}\mu\nu\ldots}\right)\right)} \tag{112}$$

and that

$${}_{\mathrm{R}}U_{\mathrm{kl}\mu\nu\ldots}\equiv\frac{\sigma\left({}_{\mathrm{R}}U_{\mathrm{kl}\mu\nu\ldots}\right)}{{}^{1/2}\left(p\left({}_{\mathrm{R}}U_{\mathrm{kl}\mu\nu\ldots}\right)\cap\left(1_{\mathrm{kl}\mu\nu\ldots}-p\left({}_{\mathrm{R}}U_{\mathrm{kl}\mu\nu\ldots}\right)\right)\right)} \tag{113}$$

The standard deviation of a second-rank tensor, denoted as $\sigma\left({}_{\mathrm{R}}U_{\mathrm{kl}\mu\nu\ldots}\right)$, would follow as

$$\begin{aligned}
&\sigma\left({}_{\mathrm{R}}U_{\mathrm{kl}\mu\nu\ldots}\right)\\
&\equiv^{1/2}\left({}_{\mathrm{R}}U_{\mathrm{kl}\mu\nu\ldots}\cap{}_{\mathrm{R}}U_{\mathrm{kl}\mu\nu\ldots}\cap p\left({}_{\mathrm{R}}U_{\mathrm{kl}\mu\nu\ldots}\right)\cap\left(\left(1_{\mathrm{kl}\mu\nu\ldots}-p\left({}_{\mathrm{R}}U_{\mathrm{kl}\mu\nu\ldots}\right)\right)\right)\right)\\
&\equiv\sqrt[2]{\left({}_{\mathrm{R}}U_{\mathrm{kl}\mu\nu\ldots}\cap{}_{\mathrm{R}}U_{\mathrm{kl}\mu\nu\ldots}\cap p\left({}_{\mathrm{R}}U_{\mathrm{kl}\mu\nu\ldots}\right)\cap\left(\left(1_{\mathrm{kl}\mu\nu\ldots}-p\left({}_{\mathrm{R}}U_{\mathrm{kl}\mu\nu\ldots}\right)\right)\right)\right)}
\end{aligned}\tag{114}$$

2.6.21. The co-variance of two tensors

Definition 2.35 (The co-variance of two tensors).

Let ${}_{\mathrm{R}}\mathrm{U}_{\mu\nu}$ denote a second-rank co-variant tensor. Let p(${}_{\mathrm{R}}\mathrm{U}_{\mu\nu}$) denote the probability of a tensor ${}_{\mathrm{R}}\mathrm{U}_{\mu\nu}$. According to equation 92, the probability of a tensor ${}_{\mathrm{R}}\mathrm{U}_{\mu\nu}$ is defined as p(${}_{\mathrm{R}}\mathrm{U}_{\mu\nu}$). Let ${}_{\mathrm{R}}\mathrm{W}_{\mu\nu}$ denote a second-rank co-variant tensor. Let p(${}_{\mathrm{R}}\mathrm{W}_{\mu\nu}$) denote the probability of a tensor ${}_{\mathrm{R}}\mathrm{W}_{\mu\nu}$ (see equation 92). Let p(${}_{\mathrm{R}}\mathrm{U}_{\mu\nu}$, ${}_{\mathrm{R}}\mathrm{W}_{\mu\nu}$) denote the probability of a joint tensor of the two tensors ${}_{\mathrm{R}}\mathrm{U}_{\mu\nu}$ and ${}_{\mathrm{R}}\mathrm{W}_{\mu\nu}$.The co-variance of the two tensors ${}_{\mathrm{R}}\mathrm{U}_{\mu\nu}$ and ${}_{\mathrm{R}}\mathrm{W}_{\mu\nu}$, denoted as $\sigma\left({}_{\mathrm{R}}U_{\mu\nu\ldots},{}_{\mathrm{R}}W_{\mu\nu\ldots}\right)$, is defined as

$$\begin{aligned}
&\sigma\left({}_{\mathrm{R}}U_{\mu\nu},{}_{\mathrm{R}}W_{\mu\nu}\right)\\
&\equiv E\left({}_{\mathrm{R}}U_{\mu\nu},{}_{\mathrm{R}}W_{\mu\nu}\right)-\left(E\left({}_{\mathrm{R}}U_{\mu\nu}\right)\times E\left({}_{\mathrm{R}}W_{\mu\nu}\right)\right)\\
&\equiv\left(p\left({}_{\mathrm{R}}U_{\mu\nu},{}_{\mathrm{R}}W_{\mu\nu}\right)\cap{}_{\mathrm{R}}U_{\mu\nu}\cap{}_{\mathrm{R}}W_{\mu\nu}\right)\\
&-\left(p\left({}_{\mathrm{R}}U_{\mu\nu}\right)\cap{}_{\mathrm{R}}U_{\mu\nu}\cap p\left({}_{\mathrm{R}}W_{\mu\nu}\right)\cap{}_{\mathrm{R}}W_{\mu\nu}\right)\\
&\equiv{}_{\mathrm{R}}U_{\mu\nu}\cap{}_{\mathrm{R}}W_{\mu\nu}\cap\left(p\left({}_{\mathrm{R}}U_{\mu\nu},{}_{\mathrm{R}}W_{\mu\nu}\right)-\left(p\left({}_{\mathrm{R}}U_{\mu\nu}\right)\times p\left({}_{\mathrm{R}}W_{\mu\nu}\right)\right)\right)
\end{aligned}\tag{115}$$

From equation 115 follows that

$${}_{\mathrm{R}}U_{\mu\nu}\cap{}_{\mathrm{R}}W_{\mu\nu}\equiv\frac{\sigma\left({}_{\mathrm{R}}U_{\mu\nu},{}_{\mathrm{R}}W_{\mu\nu}\right)}{\left(p\left({}_{\mathrm{R}}U_{\mu\nu},{}_{\mathrm{R}}W_{\mu\nu}\right)-\left(p\left({}_{\mathrm{R}}U_{\mu\nu}\right)\times p\left({}_{\mathrm{R}}W_{\mu\nu}\right)\right)\right)}\tag{116}$$

Let ${}_{\mathrm{R}}\mathrm{U}_{\mathrm{kl}\mu\nu\ldots}$ denote a co-variant n-index rank tensor. Furthermore, let p(${}_{\mathrm{R}}\mathrm{U}_{\mathrm{kl}\mu\nu\ldots}$) denote the probability of a co-variant n-index rank tensor ${}_{\mathrm{R}}\mathrm{U}_{\mathrm{kl}\mu\nu\ldots}$. According to equation 92, the probability of a co-variant n-index rank tensor ${}_{\mathrm{R}}\mathrm{U}_{\mathrm{kl}\mu\nu\ldots}$ is defined as p(${}_{\mathrm{R}}\mathrm{U}_{\mathrm{kl}\mu\nu\ldots}$). Let ${}_{\mathrm{R}}\mathrm{W}_{\mathrm{kl}\mu\nu\ldots}$ denote a co-variant n-index rank tensor. Let p(${}_{\mathrm{R}}\mathrm{W}_{\mathrm{kl}\mu\nu\ldots}$) denote the probability of this co-variant n-index rank tensor ${}_{\mathrm{R}}\mathrm{W}_{\mathrm{kl}\mu\nu\ldots}$ (see equation 92). Let p(${}_{\mathrm{R}}\mathrm{U}_{\mathrm{kl}\mu\nu\ldots}$, ${}_{\mathrm{R}}\mathrm{W}_{\mathrm{kl}\mu\nu\ldots}$) denote the probability of a joint tensor of the two co-variant n-index rank tensors ${}_{\mathrm{R}}\mathrm{U}_{\mathrm{kl}\mu\nu\ldots}$ and ${}_{\mathrm{R}}\mathrm{W}_{\mathrm{kl}\mu\nu\ldots}$.The co-variance of the two co-variant n-index rank tensor ${}_{\mathrm{R}}\mathrm{U}_{\mathrm{kl}\mu\nu\ldots}$ and ${}_{\mathrm{R}}\mathrm{W}_{\mathrm{kl}\mu\nu\ldots}$, denoted as $\sigma\left({}_{\mathrm{R}}U_{\mathrm{kl}\mu\nu\ldots},{}_{\mathrm{R}}W_{\mathrm{kl}\mu\nu\ldots}\right)$, is defined as

$$\begin{aligned}
&\sigma\left({}_{\mathrm{R}}U_{\mathrm{kl}\mu\nu\ldots},{}_{\mathrm{R}}W_{\mathrm{kl}\mu\nu\ldots}\right)\\
&\equiv E\left({}_{\mathrm{R}}U_{\mathrm{kl}\mu\nu\ldots},{}_{\mathrm{R}}W_{\mathrm{kl}\mu\nu\ldots}\right)-\left(E\left({}_{\mathrm{R}}U_{\mathrm{kl}\mu\nu\ldots}\right)\times E\left({}_{\mathrm{R}}W_{\mathrm{kl}\mu\nu\ldots}\right)\right)\\
&\equiv\left(p\left({}_{\mathrm{R}}U_{\mathrm{kl}\mu\nu\ldots},{}_{\mathrm{R}}W_{\mathrm{kl}\mu\nu\ldots}\right)\cap{}_{\mathrm{R}}U_{\mathrm{kl}\mu\nu\ldots}\cap{}_{\mathrm{R}}W_{\mathrm{kl}\mu\nu\ldots}\right)\\
&-\left(p\left({}_{\mathrm{R}}U_{\mathrm{kl}\mu\nu\ldots}\right)\cap{}_{\mathrm{R}}U_{\mathrm{kl}\mu\nu\ldots}\cap p\left({}_{\mathrm{R}}W_{\mathrm{kl}\mu\nu\ldots}\right)\cap{}_{\mathrm{R}}W_{\mathrm{kl}\mu\nu\ldots}\right)\\
&\equiv{}_{\mathrm{R}}U_{\mathrm{kl}\mu\nu\ldots}\cap{}_{\mathrm{R}}W_{\mathrm{kl}\mu\nu\ldots}\cap\left(p\left({}_{\mathrm{R}}U_{\mathrm{kl}\mu\nu\ldots},{}_{\mathrm{R}}W_{\mathrm{kl}\mu\nu\ldots}\right)-\left(p\left({}_{\mathrm{R}}U_{\mathrm{kl}\mu\nu\ldots}\right)\times p\left({}_{\mathrm{R}}W_{\mathrm{kl}\mu\nu\ldots}\right)\right)\right)
\end{aligned}\tag{117}$$

From equation 117 follows that

$$
{}_{\mathrm{R}}U_{\mathrm{kl}\mu\nu\ldots} \cap {}_{\mathrm{R}}W_{\mathrm{kl}\mu\nu\ldots} \equiv \frac{\sigma\left({}_{\mathrm{R}}U_{\mathrm{kl}\mu\nu\ldots}, {}_{\mathrm{R}}W_{\mathrm{kl}\mu\nu\ldots}\right)}{\left(p\left({}_{\mathrm{R}}U_{\mathrm{kl}\mu\nu\ldots}, {}_{\mathrm{R}}W_{\mathrm{kl}\mu\nu\ldots}\right) - \left(p\left({}_{\mathrm{R}}U_{\mathrm{kl}\mu\nu\ldots}\right) \times p\left({}_{\mathrm{R}}W_{\mathrm{kl}\mu\nu\ldots}\right)\right)\right)} \tag{118}
$$

*2.7. **Einstein's theory of special relativity***

Definition 2.36 (Energy $_{R}E_{t}$ and Matter $_{R}M_{t}$).

The equivalence of matter ($_{R}M_{t}$) and energy ($_{R}E_{t}$) lies at the core of today's physics and has been described by Einstein as follows:

"Gibt ein Körper die Energie L in Form von Strahlung ab, so verkleinert sich seine Masse um L/V^2 ... Die Masse eines Körpers ist ein Maß für dessen Energieinhalt "

(Einstein, 1905c)

In general it is

$$_{R}M_{t} \equiv \frac{_{R}E_{t}}{c^2} \tag{119}$$

(Einstein, 1905c)

where $_{R}M_{t}$ is the relativistic(Tolman, 1912) matter or matter as given from the point of view of a stationary observer R, $_{R}E_{t}$ is the total or relativistic energy of a system, of an entity et cetera, as associated with matter, c is the speed of the light in vacuum and t is the Bernoulli trial or (period of) (space-) time. In other words, Einstein is demanding the equivalence of matter and energy as the most important upshot of his special theory of relativity. "Eines der wichtigsten Resultate der Relativitätstheorie ist die Erkenntnis, daß jegliche Energie E eine ihr proportionale Trägheit (E/c^2) besitzt". (Einstein, 1912b) However, at least one main theoretical questions is surrounding Einstein's equation 119. How we ought to understand the notion of mass and matter, and that matter and energy are in some sense equivalent, may be the focus of following lines. Although it is far less common today, one still should refer to Einstein's understanding of matter and gravitational field again. Einstein is writing:

"Wir unterscheiden im folgenden zwischen 'Gravitationsfeld'und 'Materie', in dem Sinne, daß

alles außer dem Gravitationsfeld als '**Materie**'bezeichnet wird, also nicht nur

die **'Materie'im üblichen Sinne**, sondern auch das **elektromagnetische Feld**. "

(Einstein, 1916, p. 802/803)

Firstly. Everything but the gravitational field is matter, there is no third between matter and gravitational field, a third is not given, **tertium non datur**. Secondly. Matter, from the point of view of a stationary observer R, includes not only matter in the ordinary sense, but the electromagnetic field as well (Einstein, 1916, p. 802/803). Finally, one consequential relationship is necessary to mention. "Da Masse und Energie nach den Ergebnissen der speziellen Relativitätstheorie das Gleiche sind und die Energie formal durch den symmetrischen Energietensor (Tμv) beschrieben wird, so besagt dies, daß das G-Geld [gravitational field, author] durch den Energietensor der Materie bedingt und bestimmt ist "(Einstein, 1918b). Matter or energy is the cause of the gravitational field. However, is this relationship valid vice versa to?

Definition 2.37 (Time $_{R}t_{t}$ and gravitational field $_{R}g_{t}$).

The fundamental relationship between gravitational field $_{R}g_{t}$ from the point of view of the stationary observer R and time $_{R}t_{t}$ from the point of view of the same stationary observer R is determined(Barukčić, 2011, 2013, 2016b, Barukčić, 2016b) by the equation

$$ _{R}g_{t} \equiv \frac{_{R}t_{t}}{c^{2}} \tag{120}$$

and from the point of view of a co-moving observer 0 by the equation

$$ _{0}g_{t} \equiv \frac{_{0}t_{t}}{c^{2}} \tag{121}$$

Next we define(Barukčić, 2011, Barukčić, 2016b) the following mathematical identities related to time, to which a concrete physical meaning would have to be attached in the following of further development.

$$ _{W}t_{t} \equiv v \times c \times {_{R}g_{t}} \tag{122}$$

In general, it is

$$ _{W}t_{t}^{2} \equiv \left(v \times c \times {_{R}g_{t}}\right)^{2} \equiv {_{R}t_{t}}^{2} - {_{0}t_{t}}^{2} \tag{123}$$

and

$$ _{W}g_{t} \equiv \frac{_{W}t_{t}}{c^{2}} \tag{124}$$

As such (see equation 123), it is a logical step to consider that

$$ _{R}g_{t} \equiv {_{0}g_{t}} + {_{W}g_{t}} \tag{125}$$

I should like to take this opportunity to express once again the possibility that $_{W}g_{t}$ itself might represent something similar to the gravitational waves. Let the mathematical identity $_{K}t_{t}$ be defined as follows.

$$ _{K}t_{t} \equiv \frac{_{W}t_{t} \times {_{W}t_{t}}}{_{R}t_{t}} \equiv \frac{_{W}t_{t}}{_{R}t_{t}} \times {_{W}t_{t}} \equiv \frac{\left(v \times c \times {_{R}g_{t}}\right)^{2}}{c^{2} \times {_{R}g_{t}}} \equiv v^{2} \times {_{R}g_{t}} \tag{126}$$

The notion $_{K}t_{t}$ might indicate the time as determined by the relativistic kinetic energy $_{K}E_{t}$. Let the mathematical identity $_{P}t_{t}$ be defined as follows.

$$ _{P}t_{t} \equiv \frac{_{0}t_{t} \times {_{0}t_{t}}}{_{R}t_{t}} \equiv \frac{_{0}t_{t}}{_{R}t_{t}} \times {_{0}t_{t}} \equiv \left(\sqrt{1-\frac{v^{2}}{c^{2}}}\right) \times {_{0}t_{t}} \tag{127}$$

The notion ${}_{\mathrm{P}}\mathrm{t}_{\mathrm{t}}$ might indicate the time as determined by the relativistic potential energy ${}_{\mathrm{P}}\mathrm{E}_{\mathrm{t}}$. In general, it is necessary to consider that,

$$ {}_{\mathrm{R}}t_{\mathrm{t}} \equiv {}_{\mathrm{P}}t_{\mathrm{t}} + {}_{\mathrm{K}}t_{\mathrm{t}} \tag{128} $$

Furthermore, the following identities are defined.

$$ {}_{\mathrm{K}}g_{\mathrm{t}} \equiv \frac{{}_{\mathrm{K}}t_{\mathrm{t}}}{c^2} \tag{129} $$

$$ {}_{\mathrm{P}}g_{\mathrm{t}} \equiv \frac{{}_{\mathrm{P}}t_{\mathrm{t}}}{c^2} \tag{130} $$

The identity ${}_{\mathrm{Kred}}\mathrm{t}_{\mathrm{t}}$ is defined as

$$ {}_{\mathrm{Kred}}t_{\mathrm{t}} \equiv v \times {}_{\mathrm{R}}g_{\mathrm{t}} \tag{131} $$

Definition 2.38 (Space ${}_{\mathrm{R}}\mathrm{S}_{\mathrm{t}}$).

We define the general relationship

$$ {}_{\mathrm{R}}S_{\mathrm{t}} \equiv {}_{0}S_{\mathrm{t}} + {}_{0}\underline{S}_{\mathrm{t}} \equiv {}_{\mathrm{R}}U_{\mathrm{t}} \times c^2 \tag{132} $$

In case, that there are not justified reasons to doubt the correctness of Einstein's demand that all but matter is a gravitational field(Einstein, 1916, p. 802/803), we define

$$ {}_{\mathrm{R}}U_{\mathrm{t}} \equiv {}_{\mathrm{R}}M_{\mathrm{t}} + {}_{\mathrm{R}}g_{\mathrm{t}} \equiv \frac{{}_{\mathrm{R}}S_{\mathrm{t}}}{c^2} \tag{133} $$

where ${}_{\mathrm{R}}\mathrm{U}_{\mathrm{t}}$ is the mathematical identity of matter ${}_{\mathrm{R}}\mathrm{M}_{\mathrm{t}}$ and gravitational field ${}_{\mathrm{R}}\mathrm{g}_{\mathrm{t}}$, ${}_{\mathrm{R}}\mathrm{S}_{\mathrm{t}}$ is something like space and c is the speed of the light in vacuum. The following figure might illustrate this basic relationship from another point of view.

We multiply equation 133 by the term $\left(\sqrt{1-\frac{v^2}{c^2}}\right)$ where v is the relative velocity between a co-moving observer 0 and a stationary observer R. It is

$$ \left({}_{\mathrm{R}}U_{\mathrm{t}} \times \left(\sqrt{1-\frac{v^2}{c^2}}\right)\right) \equiv \left({}_{\mathrm{R}}M_{\mathrm{t}} \times \left(\sqrt{1-\frac{v^2}{c^2}}\right)\right) + \left({}_{\mathrm{R}}g_{\mathrm{t}} \times \left(\sqrt{1-\frac{v^2}{c^2}}\right)\right) \tag{134} $$

We define ${}_{0}\mathrm{U}_{\mathrm{t}}$ as

$$ {}_{0}U_{\mathrm{t}} \equiv {}_{\mathrm{R}}U_{\mathrm{t}} \times \left(\sqrt{1-\frac{v^2}{c^2}}\right) \tag{135} $$

According to Einstein, the rest-mass ${}_{0}\mathrm{m}_{\mathrm{t}}$ is given as

$$ {}_{0}m_{\mathrm{t}} \equiv {}_{\mathrm{R}}M_{\mathrm{t}} \times \left(\sqrt{1-\frac{v^2}{c^2}}\right) \tag{136} $$

We define ${}_{0}\mathrm{g}_{\mathrm{t}}$ as

$$ {}_{0}g_{\mathrm{t}} \equiv {}_{\mathrm{R}}g_{\mathrm{t}} \times \left(\sqrt{1 - \frac{v^2}{c^2}} \right) \tag{137} $$

Equation 134 as seen from the point of view of a co-moving observer 0 becomes

$$ {}_{0}U_{\mathrm{t}} \equiv {}_{0}m_{\mathrm{t}} + {}_{0}g_{\mathrm{t}} \tag{138} $$

where ${}_{0}\mathrm{m}_{\mathrm{t}}$ indicates the rest mass as determined by the co-moving observer, ${}_{0}\mathrm{g}_{\mathrm{t}}$ is the gravitational field as determined by the co-moving observer and ${}_{0}\mathrm{U}_{\mathrm{t}}$ is the unity and the 'struggle' of both.

2.8. *Einstein's general theory or relativity*

Definition 2.39 (The Einstein field equations). *The Einstein field equations (Einstein, 1915, 1916, 1917, 1935, Einstein and de Sitter, 1932) describe the relationship between the presence of matter (represented by the stress-energy tensor* $\left(\left(\frac{4\times 2\times \pi\times \gamma}{c^4}\right)\times T_{\mu\nu}\right)$ *in a given region of spacetime and the curvature in that region by the equation*

$$R_{\mu\nu} - \left(\left(\frac{R}{2}\right)\times g_{\mu\nu}\right) + (\Lambda\times g_{\mu\nu}) \equiv \left(\frac{4\times 2\times \pi\times \gamma}{c^4}\right)\times T_{\mu\nu} \equiv E_{\mu\nu} \tag{139}$$

*where $R_{\mu\nu}$ is the Ricci tensor (Ricci and Levi-Civita, 1900) of 'Einstein's general theory of relativity' (Einstein, 1916), R is the Ricci scalar, the trace of the Ricci curvature tensor with respect to the metric and equally the simplest curvature invariant of a Riemannian manifold, Λ is the Einstein's cosmological (Barukčić, 2015a, Einstein, 1917) constant, $\underline{\Lambda}$ is the "**anti cosmological constant**" (Barukčić, 2015a), $g_{\mu\nu}$ is the metric tensor of Einstein's general theory of relativity, $G_{\mu\nu}$ is Einstein's curvature tensor, $\underline{G}_{\mu\nu}$ is the "**anti tensor**" (Barukčić, 2016b) of Einstein's curvature tensor, $E_{\mu\nu}$ is the stress-energy tensor of energy, $\underline{E}_{\mu\nu}$ is the tensor of non-energy, the anti-tensor of the stress-energy tensor of energy, $a_{\mu\nu}$, $b_{\mu\nu}$, $c_{\mu\nu}$ and $d_{\mu\nu}$ denote the four basic fields of nature were $a_{\mu\nu}$ is the stress-energy tensor of ordinary matter, $b_{\mu\nu}$ is the stress-energy tensor of the electromagnetic field, c is the speed of the light in vacuum, γ is Newton's gravitational "**constant**" (Barukčić, 2016b, Barukčić, 2015a,b, 2016a), π is Archimedes constant pi.*

Table 3 may provide a more detailed and preliminary overview of the definitions (Barukčić, 2016a,b) before.

		Curvature YES	Curvature NO	
Momentum	YES	$a_{\mu\nu}$	$b_{\mu\nu} \equiv (c_{\mu\nu} + \Lambda\times g_{\mu\nu})$	$\frac{8\times \pi\times \gamma}{c^4\times D}\times g_{\mu\nu} \equiv \left(\frac{R}{D}-\frac{R}{2}+\Lambda\right)\times g_{\mu\nu}$
	NO	$c_{\mu\nu} \equiv (b_{\mu\nu} - \Lambda\times g_{\mu\nu})$	$d_{\mu\nu} \equiv (\frac{R}{2}\times g_{\mu\nu} - b_{\mu\nu})$	$\left(\frac{R}{2}-\Lambda\right)\times g_{\mu\nu}$
		$G_{\mu\nu} \equiv \left(\frac{R}{D}-\frac{R}{2}\right)\times g_{\mu\nu}$	$\frac{R}{2}\times g_{\mu\nu}$	$R_{\mu\nu} \equiv \frac{R}{D}\times g_{\mu\nu}$

Table 3. Wherever our eyes reaches, dust and stones and **nothing, who knows, who owns**.

From Einstein's specific point of view, two wings are necessary to get to the core of the relationship between matter and gravitational field, just as two wings are essential for a bird that conquers the air.

We are quite privileged to consider in detail that

$$\underbrace{\left(\frac{R}{D}\times g_{\mu\nu}\right)-\left(\left(\frac{R}{2}\right)\times g_{\mu\nu}\right)+\left(\Lambda\times g_{\mu\nu}\right)}_{the\ left-hand\ side}\equiv\underbrace{\left(\frac{4\times 2\times\pi\times\gamma\times T}{c^4\times D}\right)\times g_{\mu\nu}}_{the\ right-hand\ side} \tag{140}$$

while $R_{\mu\nu}\equiv a_{\mu\nu}+b_{\mu\nu}+c_{\mu\nu}+d_{\mu\nu}$ and the

> "... one wing ... is made of fine marble (**left side of the equation**) ...
>
> the other wing ... is built of low-grade wood (**right side of equation**).
>
> The phenomenological representation of matter is, in fact, only a crude substitute for a representation which would do justice to all known properties of matter. "
>
> (Einstein, 1936, p. 370)

Taken together, the n^th index, D-dimensional Einstein's gravitational field equations (Barukčić, 2020c) follow as

$$\underbrace{\left(\frac{R}{D}\times g_{\mu\nu\pi\rho\,\dots}\right)-\left(\left(\frac{R}{2}\right)\times g_{\mu\nu\pi\rho\,\dots}\right)+\left(\Lambda\times g_{\mu\nu\pi\rho\,\dots}\right)}_{(local)\ space-time\ curvature}\equiv\underbrace{\left(\frac{4\times 2\times\pi\times\gamma\times T}{c^4\times D}\right)\times g_{\mu\nu\pi\rho\,\dots}}_{(local)\ energy\ and\ momentum} \tag{141}$$

In general, the metric field (responsible for gravitational-inertial properties of bodies) on the left-hand side of Einstein's field equations, is completely determined by a tensorial but non-geometrical phenomenological representation of matter on the right-hand side. Einstein himself had a very differentiated view of these two sides of his field equations. In point of fact, the left part of the Einstein field equations (the Einstein tensor) is taken by Einstein as fine marble because of its geometrical nature, whereas the right side of the equations is lacking similar geometric significance and was degraded by Einstein himself to low-grade wood, the need for geometrical unification follows at least from such an asymmetrical state of affairs.

> "The mind striving after unification of the theory cannot be satisfied that two fields should exist which, by their nature, are quite independent. A mathematically unified field theory is sought in which the gravitational field and the electromagnetic field are interpreted only as different components or manifestations of the same uniform field ... The gravitational theory ... should be generalized so that it includes the laws of the electromagnetic field.
>
> (Einstein, 1923a, p. 489)

An incorporation of electromagnetism and of other fields into spacetime geometry is desirable. In point of fact, a striving toward unification and simplification of the premises and of Einstein's general theory of relativity as a whole is necessary.

Definition 2.40 (The stress-energy tensor of the electromagnetic field). *A completely geometrized, co-variant stress-energy tensor of the electromagnetic field expressed under conditions of D = 4 space-time dimensions has already been published (see theorem 3.1, equation 80; Barukčić, 2020a, p. 157). The trace-less, symmetric stress-energy tensor of the (source-free) electromagnetic field, denoted by $b_{\mu\nu}$, is expressed more compactly in a coordinate-independent way as*

$$b_{\mu\nu} \equiv \left(\frac{1}{4\times\pi} \times \left(\left(F_{\mu\ c} \times F_{\nu}{}^{c}\right) + \left(\frac{1}{4} \times g_{\mu\nu} \times F_{de} \times F^{de}\right)\right)\right) \tag{142}$$

*(see Lehmkuhl, 2011, p. 13) or (**depending upon metric signature**) as*

$$b_{\mu\nu} \equiv \left(\frac{1}{4\times\pi} \times \left(\left(F_{\mu\ c} \times F_{\nu\ d} \times g^{cd}\right) - \left(\frac{1}{4} \times g_{\mu\nu} \times F_{de} \times F^{de}\right)\right)\right) \tag{143}$$

(see Hughston and Tod, 1990, p. 38).

Definition 2.41 (The stress-energy tensor of ordinary matter $\mathbf{a}_{\mu\nu}$). *Howard Georgi and Sheldon Glashow (Georgi and Glashow, 1974) proposed in 1974 the first Grand Unified Theory (Buras et al., 1978). Grand Unified Theory (GUT) models predict the unification of the electromagnetic, the weak, and the strong forces into a single force. However, it appears to be more appropriate to unify the strong nuclear force and the weak nuclear force into an ordinary force. The matter as associated with an ordinary force can be calculated very precisely. Under conditions of Einstein's general (Einstein, 1915, 1916, 1917, 1935, Einstein and de Sitter, 1932) theory of relativity, **the stress-energy tensor of ordinary matter $\mathbf{a}_{\mu\nu}$** which **is expected to unify the strong nuclear force and the weak nuclear force into an ordinary force** is defined / derived / determined as*

$$\begin{aligned} a_{\mu\nu} &\equiv \left(\left(\frac{4\times 2\times\pi\times\gamma}{c^4}\right) \times T_{\mu\nu}\right) - b_{\mu\nu} \\ &\equiv G_{\mu\nu} + \left(\Lambda \times g_{\mu\nu}\right) - b_{\mu\nu} \\ &\equiv R_{\mu\nu} - \left(R \times g_{\mu\nu}\right) + \left(\Lambda \times g_{\mu\nu}\right) + d_{\mu\nu} \\ &\equiv (E-b) \times g_{\mu\nu} \\ &\equiv (G-c) \times g_{\mu\nu} \\ &\equiv a \times g_{\mu\nu} \end{aligned} \tag{144}$$

or

$$\begin{aligned} a_{\mu\nu} \equiv R_{\mu\nu} - \left(\left(\frac{R}{2}\right) \times g_{\mu\nu}\right) + \left(\Lambda \times g_{\mu\nu}\right) - \\ \left(\frac{1}{4\times\pi} \times \left(\left(F_{\mu\ c} \times F_{\nu\ d} \times g^{cd}\right) + \left(\frac{1}{4} \times g_{\mu\nu} \times F_{de} \times F^{de}\right)\right)\right) \end{aligned} \tag{145}$$

From our present point of view we can expect that there are conditions where

$$\begin{aligned} a_{\mu\nu} &\equiv \left(\left(\frac{4 \times 2 \times \pi \times \gamma}{c^4} \right) \times T_{\mu\nu} \right) - b_{\mu\nu} \\ &\equiv \left(\left(\frac{R}{D} - \frac{R}{2} + \Lambda \right) - \left(\frac{(4+D) \times F_1}{4 \times \pi \times 4 \times D} \right) \right) \times g_{\mu\nu} \end{aligned} \tag{146}$$

where F_1 is Lorenz invariant.

Definition 2.42 (**The 4-index D dimensional a $_{\mathbf{kl}\mu\nu}$**). *The 4-index D dimensional a $_{kl\mu\nu}$ is defined as:*

$$\begin{aligned} a_{kl\mu\nu} &\equiv (E-b) \times g_{kl\mu\nu} \\ &\equiv (G-c) \times g_{kl\mu\nu} \\ &\equiv a \times g_{kl\mu\nu} \end{aligned} \tag{147}$$

Definition 2.43 (**The n-index D dimensional a $_{\mathbf{kl}\mu\nu\ldots}$**). *The n-index D dimensional a $_{kl\mu\nu\ldots}$ is defined as:*

$$\begin{aligned} a_{kl\mu\nu\ldots} &\equiv (E-b) \times g_{kl\mu\nu\ldots} \\ &\equiv (G-c) \times g_{kl\mu\nu\ldots} \\ &\equiv a \times g_{kl\mu\nu\ldots} \end{aligned} \tag{148}$$

Definition 2.44 (**Ricci scalar R**). *Under conditions of Einstein's general (Einstein, 1915, 1916, 1917, 1935, Einstein and de Sitter, 1932) theory of relativity, the Ricci scalar curvature R as the trace of the Ricci curvature tensor $R_{\mu\nu}$ with respect to the metric is determined at each point in space-time by lamda Λ and anti-lamda (Barukčić, 2015a) $\underline{\Lambda}$ as*

$$R \equiv g^{\mu\nu} \times R_{\mu\nu} \equiv (\Lambda) + (\underline{\Lambda}) \equiv D \times S \tag{149}$$

where D is the number of space-time dimension and $S \equiv \left(\frac{R}{D}\right)$ (see theorem 3.16, equation 369). A Ricci scalar curvature R which is positive at a certain point indicates that the volume of a small ball about the point has smaller volume than a ball of the same radius in Euclidean space. In other words, the density of space varies. In contrast to this, a Ricci scalar curvature R which is negative at a certain point indicates that the volume of a small ball is larger than it would be in Euclidean space. In general, it is (see Barukčić, 2015a)

$$R \times g_{\mu\nu} \equiv (\Lambda \times g_{\mu\nu}) + (\underline{\Lambda} \times g_{\mu\nu}) \tag{150}$$

or

$$R \equiv (\Lambda) + (\underline{\Lambda}) \tag{151}$$

The cosmological constant can also be written algebraically as part of the stress–energy tensor, a second order tensor as the source of gravity (energy density).

Definition 2.45 (**Ricci tensor $\mathbf{R}_{\mu\nu}$**). *The Ricci tensor $R_{\mu\nu}$ is a geometric object which has been developed by Gregorio Ricci-Curbastro (1853 – 1925) (Ricci and Levi-Civita, 1900) and is able to measure of the degree to which a certain geometry of a given metric differs from that of ordinary Euclidean space. In this publication, let $a_{\mu\nu}$, $b_{\mu\nu}$, $c_{\mu\nu}$ and $d_{\mu\nu}$ denote the covariant second rank tensors* *of the four basic fields of nature were* $a_{\mu\nu} \equiv {}_{f}a^{2} \times g_{\mu\nu}$ *is the stress-energy tensor of ordinary matter,* $b_{\mu\nu} \equiv {}_{f}b^{2} \times g_{\mu\nu}$ *is the stress-energy tensor of the electromagnetic field,* $c_{\mu\nu} \equiv c^{2} \times g_{\mu\nu}$ *is the tensor of the gravitational field and* $d_{\mu\nu} \equiv {}_{f}d^{2} \times g_{\mu\nu}$ *is the tensor of gravitational waves. The Ricci tensor* $R_{\mu\nu}$ *of 'Einstein's general theory of relativity' (Einstein, 1916) is determined by the stress-energy tensor* $\left(\left(\frac{4 \times 2 \times \pi \times \gamma}{c^{4}}\right) \times T_{\mu\nu}\right)$ *and the anti stress-energy tensor* $\left(\left(\left(\frac{R}{2}\right) \times g_{\mu\nu}\right) - \left(\Lambda \times g_{\mu\nu}\right)\right)$ *as*

$$\begin{aligned} R_{\mu\nu} &\equiv \underbrace{\left(\left(\frac{4 \times 2 \times \pi \times \gamma}{c^{4}}\right) \times T_{\mu\nu}\right)}_{stress-energy\ tensor} + \underbrace{\left(\left(\left(\frac{R}{2}\right) \times g_{\mu\nu}\right) - \left(\Lambda \times g_{\mu\nu}\right)\right)}_{anti\ stress-energy\ tensor} \\ &\equiv \qquad a_{\mu\nu} + b_{\mu\nu} \qquad\qquad + c_{\mu\nu} + d_{\mu\nu} \\ &\equiv (S) \times g_{\mu\nu} \\ &\equiv \left(\frac{R}{D}\right) \times g_{\mu\nu} \end{aligned} \tag{152}$$

while S might denote a scalar.

Definition 2.46 (Laue's scalar T). *Max von Laue (1879-1960) proposed the meanwhile so called Laue scalar (Laue, 1911) (criticised by Einstein (Einstein and Grossmann, 1913)) as the contraction of the the stress–energy momentum tensor* $T_{\mu\nu}$ *denoted as T and written without subscripts or arguments. Under conditions of Einstein's general (Einstein, 1915, 1916, 1917, 1935, Einstein and de Sitter, 1932) theory of relativity, it is*

$$T \equiv g^{\mu\nu} \times T_{\mu\nu} \tag{153}$$

Taken Einstein seriously, $T_{\mu\nu}$ *"denotes the co-variant energy tensor of matter" (see Einstein, 1923b, p. 88). In other words, "Considered phenomenologically, this energy tensor is composed of that of the electromagnetic field and of matter in the narrower sense." (see Einstein, 1923b, p. 93)*

Definition 2.47 (The scalar E). *In general, we define the scalar E as*

$$\begin{aligned} E \equiv {}_{d}E_{t}^{2} &\equiv \left(\frac{8 \times \pi \times \gamma}{c^{4} \times D}\right) \times T \\ &\equiv \left(\frac{8 \times \pi \times \gamma \times T}{c^{4} \times D}\right) \\ &\equiv \left(\frac{2 \times \pi \times 4 \times \gamma \times T}{c^{4} \times D}\right) \\ &\equiv \left(\frac{h \times 4 \times \gamma \times T}{\hbar \times c^{4} \times D}\right) \\ &\equiv \left(\frac{R}{D}\right) - \left(\frac{R}{2}\right) + \Lambda \end{aligned} \tag{154}$$

where D is the space-time dimension, where c denote the speed of the light in vacuum, γ *denote Newton's gravitational "**constant**" (Barukčić, 2016b, Barukčić, 2015a,b, 2016a),* π *is the number pi and*

T denote Laue's scalar. The scalar E might correspond even to the total energy density squared of a (relativistic or quantum) system, and has the potential as such to bridge the gap between relativity theory and quantum mechanics under circumstances where the same is related or even identical with the Hamiltonian operator (squared).

Definition 2.48 (**Stress-energy and momentum tensor** $\mathbf{E}_{\mu\nu}$). *The tensor of stress-energy-momentum denoted as $E_{\mu\nu}$ is determined in detail as follows.*

$$\begin{aligned}
E_{\mu\nu} &\equiv \left(\frac{4 \times 2 \times \pi \times \gamma}{c^4 \times D}\right) \times T_{\mu\nu} \\
&\equiv R_{\mu\nu} - \left(\left(\frac{R}{2}\right) \times g_{\mu\nu}\right) + \left(\Lambda \times g_{\mu\nu}\right) \\
&\equiv \left(S - \left(\frac{R}{2}\right) + \Lambda\right) \times g_{\mu\nu} \\
&\equiv (G + \Lambda) \times g_{\mu\nu} \\
&\equiv G_{\mu\nu} + \left(\Lambda \times g_{\mu\nu}\right) \\
&\equiv R_{\mu\nu} - \underline{E}_{\mu\nu} \\
&\equiv E \times g_{\mu\nu}
\end{aligned} \tag{155}$$

while E might denote the scalar of, even something like 'energy density'. According to Einstein, it is necessary to consider that

"... a tensor, $T_{\mu\nu}$, of the second rank ... includes in itself

the energy density of the electromagnetic field

and of

ponderable matter;

we shall denote this in the following as the 'energy tensor of matter'"

(Einstein, 1923b, pp. 87/88)

Definition 2.49 (**The scalar G**)**.** *In general, we define the scalar G (Barukčić, 2020a) as*

$$
\begin{aligned}
G \equiv {}_{d}G_{t}{}^{2} &\equiv \left(\left(\frac{R}{D} \right) - \frac{R}{2} \right) \\
&\equiv \left(E + {}_{R}t_{t} - \frac{R}{2} \right) \\
&\equiv \left(E + \left(\frac{R}{2} - \Lambda \right) - \frac{R}{2} \right) \\
&\equiv E - \Lambda
\end{aligned}
\tag{156}
$$

Definition 2.50 (**Einstein's curvature tensor $\mathbf{G}_{\mu\nu}$**)**.** *Under conditions of Einstein's general (Einstein, 1915, 1916, 1917, 1935, Einstein and de Sitter, 1932) theory of relativity, the tensor of curvature denoted by $G_{\mu\nu}$ is defined/derived/determined (see Barukčić, 2020a) as follows:*

$$
\begin{aligned}
G_{\mu\nu} &\equiv R_{\mu\nu} - \left(\left(\frac{R}{2} \right) \times g_{\mu\nu} \right) \\
&\equiv \left(\frac{R}{D} \right) \times g_{\mu\nu} - \left(\left(\frac{R}{2} \right) \times g_{\mu\nu} \right) \\
&\equiv \left(\left(\frac{R}{D} \right) - \frac{R}{2} \right) \times g_{\mu\nu} \\
&\equiv a_{\mu\nu} + c_{\mu\nu} \\
&\equiv G \times g_{\mu\nu} \\
&\equiv \left(\frac{R}{D} \right) \times {}_{G}g_{\mu\nu}
\end{aligned}
\tag{157}
$$

Definition 2.51 (**The scalar $\underline{\mathbf{G}}$**)**.** *In general, we define the scalar $\underline{G}$ (see Barukčić, 2020a) as*

$$
\begin{aligned}
\underline{G} \equiv {}_{d}\underline{G}_{t}{}^{2} &\equiv \left(\left(\frac{R}{D} \right) - G \right) \\
&\equiv \left(\frac{R}{2} \right)
\end{aligned}
\tag{158}
$$

Definition 2.52 (**The scalar $\underline{\mathbf{E}}$**)**.** *In general, we define the scalar $\underline{E}$ as (see Barukčić, 2020a)*

$$
\begin{aligned}
\underline{E} \equiv {}_{d}\underline{E}_{t}{}^{2} &\equiv \left(\left(\frac{R}{D} \right) - E \right) \\
&\equiv \left(\frac{R}{2} - \Lambda \right)
\end{aligned}
\tag{159}
$$

Remark 2.1. *In the following of research, it is appropriate to prove the relationship between (1/X) and the complex conjugate of the wave function Ψ^* or the identity (1/X)$\equiv \Psi^*$.*

Definition 2.53 (**The anti Einstein's curvature tensor or the tensor of non-curvature $\underline{G}_{\mu\nu}$**). *Under conditions of Einstein's general (Einstein, 1915, 1916, 1917, 1935, Einstein and de Sitter, 1932) theory of relativity, the tensor of non-curvature is defined/derived/determined (Barukčić, 2020a) as follows:*

$$
\begin{aligned}
\underline{G}_{\mu\nu} &\equiv R_{\mu\nu} - G_{\mu\nu} \\
&\equiv R_{\mu\nu} - \left(R_{\mu\nu} - \left(\left(\frac{R}{2} \right) \times g_{\mu\nu} \right) \right) \\
&\equiv \left(\frac{R}{2} \right) \times g_{\mu\nu} \\
&\equiv b_{\mu\nu} + d_{\mu\nu} \\
&\equiv \underline{G} \times g_{\mu\nu}
\end{aligned}
\tag{160}
$$

Definition 2.54 (**The 4-index D dimensional stress-energy and momentum tensor $\mathbf{E}_{\mathbf{kl}\mu\nu}$**). *The 4-index D dimensional stress-energy-momentum tenosr denoted as $E_{kl\mu\nu}$ is determined in detail as*

$$
\begin{aligned}
E_{kl\mu\nu} &\equiv \left(\frac{8 \times \pi \times \gamma \times T}{c^4 \times D} \right) \times g_{kl\mu\nu} \\
&\equiv R_{kl\mu\nu} - \left(\left(\frac{R}{2} \right) \times g_{kl\mu\nu} \right) + \left(\Lambda \times g_{kl\mu\nu} \right) \\
&\equiv G_{kl\mu\nu} + \left(\Lambda \times g_{kl\mu\nu} \right) \\
&\equiv R_{kl\mu\nu} - \underline{E}_{kl\mu\nu} \\
&\equiv a_{kl\mu\nu} + b_{kl\mu\nu} \\
&\equiv H \times g_{kl\mu\nu} \equiv H_{kl\mu\nu} \\
&\equiv E \times g_{kl\mu\nu}
\end{aligned}
\tag{161}
$$

Definition 2.55 (**The n-index D dimensional stress-energy and momentum tensor $\mathbf{E}_{\mathbf{kl}\mu\nu\ldots}$**). *The n-index D dimensional stress-energy-momentum tenosr denoted as $E_{kl\mu\nu\ldots}$ is determined in detail as*

$$
\begin{aligned}
E_{kl\mu\nu\ldots} &\equiv \left(\frac{8 \times \pi \times \gamma \times T}{c^4 \times D} \right) \times g_{kl\mu\nu\ldots} \\
&\equiv R_{kl\mu\nu\ldots} - \left(\left(\frac{R}{2} \right) \times g_{kl\mu\nu\ldots} \right) + \left(\Lambda \times g_{kl\mu\nu\ldots} \right) \\
&\equiv G_{kl\mu\nu\ldots} + \left(\Lambda \times g_{kl\mu\nu\ldots} \right) \\
&\equiv R_{kl\mu\nu\ldots} - \underline{E}_{kl\mu\nu\ldots} \\
&\equiv a_{kl\mu\nu\ldots} + b_{kl\mu\nu\ldots} \\
&\equiv H \times g_{kl\mu\nu\ldots} \equiv H_{kl\mu\nu\ldots} \\
&\equiv E \times g_{kl\mu\nu\ldots}
\end{aligned}
\tag{162}
$$

Definition 2.56 (The tensor of non-energy $\underline{E}_{\mu\nu}$). *Under conditions of Einstein's general (Einstein, 1915, 1916, 1917, 1935, Einstein and de Sitter, 1932) theory of relativity, the tensor of non-energy or the anti tensor of the stress energy tensor is defined/derived/determined as follows:*

$$
\begin{aligned}
\underline{E}_{\mu\nu} &\equiv R_{\mu\nu} - \left(\frac{4 \times 2 \times \pi \times \gamma}{c^4}\right) \times T_{\mu\nu} \\
&\equiv \left(\left(\frac{R}{2}\right) \times g_{\mu\nu}\right) - (\Lambda \times g_{\mu\nu}) \\
&\equiv \left(\left(\frac{R}{2} - \Lambda\right) \times g_{\mu\nu}\right) \\
&\equiv c_{\mu\nu} + d_{\mu\nu} \\
&\equiv \Psi \times g_{\mu\nu} \equiv \Psi_{\mu\nu} \\
&\equiv \underline{E} \times g_{\mu\nu}
\end{aligned}
\tag{163}
$$

Definition 2.57 (The 4-index D dimensional tensor of non-energy $\underline{E}_{kl\mu\nu}$). *The 4-index D dimensional tensor (Einstein, 1915, 1916, 1917, 1935, Einstein and de Sitter, 1932) of non-energy $\underline{E}_{kl\mu\nu}$ is defined as follows:*

$$
\begin{aligned}
\underline{E}_{kl\mu\nu} &\equiv \left(\frac{R}{D} \times g_{kl\mu\nu}\right) - \left(\left(\frac{8 \times \pi \times \gamma \times T}{c^4 \times D}\right) \times g_{kl\mu\nu}\right) \\
&\equiv \left(\left(\frac{R}{2}\right) \times g_{kl\mu\nu}\right) - (\Lambda \times g_{kl\mu\nu}) \\
&\equiv \left(\left(\frac{R}{2} - \Lambda\right) \times g_{kl\mu\nu}\right) \\
&\equiv c_{kl\mu\nu} + d_{kl\mu\nu} \\
&\equiv \Psi \times g_{kl\mu\nu} \equiv \Psi_{kl\mu\nu} \\
&\equiv \underline{E} \times g_{kl\mu\nu}
\end{aligned}
\tag{164}
$$

Definition 2.58 (The n-th index D dimensional tensor of non-energy $\underline{E}_{kl\mu\nu\ldots}$). *The n-th index D dimensional tensor (Einstein, 1915, 1916, 1917, 1935, Einstein and de Sitter, 1932) of non-energy $\underline{E}_{kl\mu\nu\ldots}$ is defined as follows:*

$$
\begin{aligned}
\underline{E}_{kl\mu\nu\ldots} &\equiv \left(\frac{R}{D} \times g_{kl\mu\nu\ldots}\right) - \left(\left(\frac{8 \times \pi \times \gamma \times T}{c^4 \times D}\right) \times g_{kl\mu\nu\ldots}\right) \\
&\equiv \left(\left(\frac{R}{2}\right) \times g_{kl\mu\nu\ldots}\right) - (\Lambda \times g_{kl\mu\nu\ldots}) \\
&\equiv \left(\left(\frac{R}{2} - \Lambda\right) \times g_{kl\mu\nu\ldots}\right) \\
&\equiv c_{kl\mu\nu\ldots} + d_{kl\mu\nu\ldots} \\
&\equiv \Psi \times g_{kl\mu\nu\ldots} \equiv \Psi_{kl\mu\nu\ldots} \\
&\equiv \underline{E} \times g_{kl\mu\nu\ldots}
\end{aligned}
\tag{165}
$$

Definition 2.59 (**The 4-index D dimensional Einstein's curvature tensor $G_{kl\mu\nu}$**). *The Riemann tensor $R_{kl\mu\nu}$ does not appear explicitly in Einstein's gravitational field equations. Therefore, the question is justified whether Einstein's equation of gravitation are really the most general equations. Frédèric Moulin proposed in the year 2017 a kind of a generalized 4-index gravitational field equation which contains the Riemann curvature tensor linearly (Moulin, 2017). Moulin himself ascribed an energy-momentum to the gravitational field itself (Moulin, 2017, p. 5/8) which is not without problems. Besides of all, it is known that the Riemann curvature tensor of general relativity $R_{kl\mu\nu}$ can be split into different ways, including the Weyl conformal tensor $C_{kl\mu\nu}$ and the anti-Weyl conformal tensor $\underline{C}_{kl\mu\nu}$ or in other words the parts which involve only the Ricci tensor $R_{\mu\nu}$ the curvature scalar R. Because of these properties $\left(R_{kl\mu\nu} \equiv C_{kl\mu\nu} + \underline{C}_{kl\mu\nu}\right)$ it is possible to reformulate the famous Einstein equation. The 4-index D dimensional Einstein's curvature tensor (Einstein, 1915, 1916, 1917, 1935, Einstein and de Sitter, 1932) denoted by $G_{kl\mu\nu}$ is defined (see Barukčić, 2020a) as follows:*

$$
\begin{aligned}
G_{kl\mu\nu} &\equiv R_{kl\mu\nu} - \left(\left(\frac{R}{2}\right) \times g_{kl\mu\nu}\right)\\
&\equiv \left(\frac{R}{D}\right) \times g_{kl\mu\nu} - \left(\left(\frac{R}{2}\right) \times g_{kl\mu\nu}\right)\\
&\equiv \left(\left(\frac{R}{D}\right) - \frac{R}{2}\right) \times g_{kl\mu\nu}\\
&\equiv a_{kl\mu\nu} + c_{kl\mu\nu}\\
&\equiv G \times g_{kl\mu\nu}
\end{aligned}
\tag{166}
$$

Definition 2.60 (**The n-index D dimensional Einstein's curvature tensor $G_{kl\mu\nu\ldots}$**). *The n-index D dimensional Einstein's curvature tensor (Einstein, 1915, 1916, 1917, 1935, Einstein and de Sitter, 1932) denoted by $G_{kl\mu\nu\ldots}$ is defined (see Barukčić, 2020a) as follows:*

$$
\begin{aligned}
G_{kl\mu\nu\ldots} &\equiv R_{kl\mu\nu\ldots} - \left(\left(\frac{R}{2}\right) \times g_{kl\mu\nu\ldots}\right)\\
&\equiv \left(\frac{R}{D}\right) \times g_{kl\mu\nu\ldots} - \left(\left(\frac{R}{2}\right) \times g_{kl\mu\nu\ldots}\right)\\
&\equiv \left(\left(\frac{R}{D}\right) - \frac{R}{2}\right) \times g_{kl\mu\nu\ldots}\\
&\equiv a_{kl\mu\nu\ldots} + c_{kl\mu\nu\ldots}\\
&\equiv G \times g_{kl\mu\nu\ldots}
\end{aligned}
\tag{167}
$$

Definition 2.61 (**The 4-index D dimensional anti Einstein's curvature tensor or the tensor or non-curvature $\underline{G}_{kl\mu\nu}$**). *The 4-index D dimensional anti Einstein's curvature tensor (Einstein, 1915, 1916, 1917, 1935, Einstein and de Sitter, 1932) or the tensor of non-curvature denoted as $\underline{G}_{kl\mu\nu}$ is defined/derived/determined (Barukčić, 2020a) as follows:*

$$
\begin{aligned}
\underline{G}_{kl\mu\nu} &\equiv R_{kl\mu\nu} - G_{kl\mu\nu} \\
&\equiv R_{kl\mu\nu} - \left(R_{kl\mu\nu} - \left(\left(\frac{R}{2} \right) \times g_{kl\mu\nu} \right) \right) \\
&\equiv \left(\frac{R}{2} \right) \times g_{kl\mu\nu} \\
&\equiv b_{kl\mu\nu} + d_{kl\mu\nu} \\
&\equiv \underline{G} \times g_{kl\mu\nu}
\end{aligned}
\tag{168}
$$

Definition 2.62 (**The n-index D dimensional anti Einstein's curvature tensor or the tensor of non-curvature $\underline{\mathbf{G}}_{\mathbf{kl}\mu\nu\ldots}$**). *The n-index D dimensional anti Einstein's curvature tensor or the tensor of non-curvature denoted as $\underline{G}_{kl\mu\nu\ldots}$ is defined/derived/determined (Barukčić, 2020a) as follows:*

$$
\begin{aligned}
\underline{G}_{kl\mu\nu\ldots} &\equiv R_{kl\mu\nu\ldots} - G_{kl\mu\nu\ldots} \\
&\equiv R_{kl\mu\nu\ldots} - \left(R_{kl\mu\nu\ldots} - \left(\left(\frac{R}{2} \right) \times g_{kl\mu\nu\ldots} \right) \right) \\
&\equiv \left(\frac{R}{2} \right) \times g_{kl\mu\nu\ldots} \\
&\equiv b_{kl\mu\nu\ldots} + d_{kl\mu\nu\ldots} \\
&\equiv \underline{G} \times g_{kl\mu\nu\ldots}
\end{aligned}
\tag{169}
$$

Definition 2.63 (**The first quadratic Lorentz invariant $\mathbf{F_1}$**). *The inner product of Faraday's electromagnetic field strength tensor yields a Lorentz invariant. The Lorentz invariant does not change from one frame of reference to another. The first quadratic Lorentz invariant, denoted as F_1 is determined as*

$$
F_1 \equiv F_{kl} \times F^{kl} \tag{170}
$$

The electromagnetic field tensor $\mathrm{F_{kl}}$ has two Lorentz invariant quantities. One of the two fundamental Lorentz invariant quantities of the electromagnetic field (Escobar and Urrutia, 2014) is known be $F_{\mathrm{kl}} \times F^{\mathrm{kl}} = 2 \times \left(B^2 - E^2\right)$ where E denotes the electric E and B the magnetic field in the taken frame of reference.

Definition 2.64 (**The second quadratic Lorentz invariant $\mathbf{F_2}$**). *The second quadratic Lorentz invariant, denoted as F_2 is determined as*

$$
F_2 \equiv \varepsilon^{klmn} \times F_{kl} \times F_{mn} \tag{171}
$$

Definition 2.65 (**The tensor $\mathbf{b}_{\mu\nu}$**). *Again, the co-variant Minkowski's stress-energy tensor of the electromagnetic field, in this context denoted by $b_{\mu\nu}$, is of order two and its components can be displayed* *by a 4 × 4 matrix too. The trace of energy-momentum tensor of the electromagnetic field is known to be null. Under conditions of Einstein's general theory of relativity (Einstein, 1915, 1916, 1917, 1935, Einstein and de Sitter, 1932), the tensor $b_{\mu\nu}$ denotes the trace-less, symmetric stress-energy tensor for source-free electromagnetic field is defined in cgs-Gaussian units (**depending upon metric signature**) as*

$$b_{\mu\nu} \equiv \left(\frac{1}{4\times\pi} \times \left(\left(F_{\mu\ c} \times F_{\nu}{}^{c}\right) + \left(\frac{1}{4} \times g_{\mu\nu} \times F_{de} \times F^{de}\right)\right)\right) \tag{172}$$

(see Lehmkuhl, 2011, p. 13) and equally as

$$b_{\mu\nu} \equiv \left(\frac{1}{4\times\pi} \times \left(\left(F_{\mu\ c} \times F_{\nu\ d} \times g^{cd}\right) - \left(\frac{1}{4} \times g_{\mu\nu} \times F_{de} \times F^{de}\right)\right)\right) \tag{173}$$

(see Hughston and Tod, 1990, p. 38). The co-variant Minkowski's stress-energy tensor of the electromagnetic field is expressed under conditions of D = 4 space-time dimensions more compactly in a coordinate-independent (theorem 3.1, equation 80 Barukčić, 2020a, p. 157) form as

$$\begin{aligned} b_{\mu\nu} &\equiv \left(\frac{1}{4\times\pi} \times \left(\left(F_{\mu\ c} \times F_{\nu\ d} \times g^{cd}\right) + \left(\frac{1}{4} \times g_{\mu\nu} \times F_{de} \times F^{de}\right)\right)\right) \\ &\equiv \left(\frac{1}{4\times\pi} \times \left(\left(F_{\mu\ c} \times F^{\mu\ c}\right) + \left(\frac{F_{I}}{4}\right)\right)\right) \times g_{\mu\nu} \\ &\equiv \left(\left(\frac{R}{D}\right) - a - c - d\right) \times g_{\mu\nu} \\ &\equiv (E - a) \times g_{\mu\nu} \\ &\equiv b \times g_{\mu\nu} \end{aligned} \tag{174}$$

where F_{de} is called the (traceless) Faraday/electromagnetic/field strength tensor.

Definition 2.66 (The 4-index D dimensional stress-energy tensor of electromagnetic field $b_{kl\mu\nu}$). *The 4-index D dimensional stress-energy tensor of electromagnetic field $b_{kl\mu\nu}$ is defined as:*

$$\begin{aligned} b_{kl\mu\nu} &\equiv \left(\left(\frac{R}{D}\right) - a - c - d\right) \times g_{kl\mu\nu} \\ &\equiv (E - a) \times g_{kl\mu\nu} \\ &\equiv b \times g_{kl\mu\nu} \end{aligned} \tag{175}$$

Definition 2.67 (The n-index D dimensional stress-energy tensor of electromagnetic field $b_{kl\mu\nu\,\ldots}$). *The n-index D dimensional stress-energy tensor of electromagnetic field $b_{kl\mu\nu\,\ldots}$ is defined as:*

$$\begin{aligned} b_{kl\mu\nu\,\ldots} &\equiv \left(\left(\frac{R}{D}\right) - a - c - d\right) \times g_{kl\mu\nu\,\ldots} \\ &\equiv (E - a) \times g_{kl\mu\nu\,\ldots} \\ &\equiv b \times g_{kl\mu\nu\,\ldots} \end{aligned} \tag{176}$$

Definition 2.68 (The tensor $c_{\mu\nu}$). *Under conditions of Einstein's general (Einstein, 1915, 1916, 1917, 1935, Einstein and de Sitter, 1932) theory of relativity, the tensor of non-momentum and curvature is defined/derived/determined (Barukčić, 2020a) as follows:*

$$
\begin{aligned}
c_{\mu\nu} &\equiv b_{\mu\nu} - \left(\Lambda \times g_{\mu\nu}\right) \\
&\equiv (G-a) \times g_{\mu\nu} \\
&\equiv \left(\frac{R}{2} - \Lambda - d\right) \times g_{\mu\nu} \\
&\equiv (b-\Lambda) \times g_{\mu\nu} \\
&\equiv c \times g_{\mu\nu}
\end{aligned}
\tag{177}
$$

Definition 2.69 (The 4-index D dimensional tensor $\mathbf{c}_{\mathbf{kl}\mu\nu}$). *The 4-index D dimensional $c_{kl\mu\nu}$ is defined as:*

$$
\begin{aligned}
c_{kl\mu\nu} &\equiv (G-a) \times g_{kl\mu\nu} \\
&\equiv \left(\frac{R}{2} - \Lambda - d\right) \times g_{kl\mu\nu} \\
&\equiv (b-\Lambda) \times g_{kl\mu\nu} \\
&\equiv c \times g_{kl\mu\nu}
\end{aligned}
\tag{178}
$$

Definition 2.70 (The n-index D dimensional tensor $\mathbf{c}_{\mathbf{kl}\mu\nu\ldots}$). *The n-index D dimensional $c_{kl\mu\nu\ldots}$ is defined as:*

$$
\begin{aligned}
c_{kl\mu\nu\ldots} &\equiv (G-a) \times g_{kl\mu\nu\ldots} \\
&\equiv \left(\frac{R}{2} - \Lambda - d\right) \times g_{kl\mu\nu\ldots} \\
&\equiv (b-\Lambda) \times g_{kl\mu\nu\ldots} \\
&\equiv c \times g_{kl\mu\nu\ldots}
\end{aligned}
\tag{179}
$$

Definition 2.71 (The tensor of neither curvature nor momentum $\mathbf{d}_{\mu\nu}$). *Under conditions of Einstein's general (Einstein, 1915, 1916, 1917, 1935, Einstein and de Sitter, 1932) theory of relativity, the tensor of neither curvature nor momentum is defined/derived/determined (Barukčić, 2020a) as follows:*

$$
\begin{aligned}
d_{\mu\nu} &\equiv \left(\left(\frac{R}{2} \right) \times g_{\mu\nu} \right) - b_{\mu\nu} \\
&\equiv \left(\left(\frac{R}{2} \right) \times g_{\mu\nu} \right) - (\Lambda \times g_{\mu\nu}) - c_{\mu\nu} \\
&\equiv \left(\frac{\left(\left(\frac{R}{D} \right) \times D \right)}{2} - b \right) \times g_{\mu\nu} \\
&\equiv \left(\frac{\left(\left(\frac{R}{D} \right) \times D \right)}{2} - \Lambda - c \right) \times g_{\mu\nu} \\
&\equiv \frac{R}{D} \times {}_{gw}g_{\mu\nu} \\
&\equiv d \times g_{\mu\nu}
\end{aligned}
\tag{180}
$$

There may exist circumstances where this tensor might indicate something like the density of gravitational waves. In detail, it is

$$
d_{\mu\nu} \equiv \frac{R}{D} \times {}_{gw}g_{\mu\nu} \equiv \left(\left(\frac{R}{2} \right) \times g_{\mu\nu} \right) - \left(\frac{1}{4 \times \pi} \times \left(\left(F_{\mu\, c} \times F_{\nu\, d} \times g^{cd} \right) + \left(\frac{1}{4} \times g_{\mu\nu} \times F_{de} \times F^{de} \right) \right) \right)
\tag{181}
$$

Under these circumstances, the metric tensor of the gravitational waves ${}_{gw}g_{\mu\nu}$ *would follow as*

$$
{}_{d}g_{\mu\nu} \equiv {}_{gw}g_{\mu\nu} \equiv \frac{D}{R} \times \left(\left(\left(\frac{R}{2} \right) \times g_{\mu\nu} \right) - \left(\frac{1}{4 \times \pi} \times \left(\left(F_{\mu\, c} \times F_{\nu\, d} \times g^{cd} \right) + \left(\frac{1}{4} \times g_{\mu\nu} \times F_{de} \times F^{de} \right) \right) \right) \right)
\tag{182}
$$

The cosmic microwave background (CMBR) radiation (Penzias and Wilson, 1965) is an electromagnetic radiation which is part of the tensor $b_{\mu\nu}$.

Definition 2.72 (The 4-index D dimensional d $_{\mathbf{kl}\mu\nu}$). *The 4-index D dimensional* $d_{kl\mu\nu}$ *is defined as:*

$$
\begin{aligned}
d_{kl\mu\nu} &\equiv \left(\frac{\left(\left(\frac{R}{D} \right) \times D \right)}{2} - b \right) \times g_{kl\mu\nu} \\
&\equiv \left(\frac{\left(\left(\frac{R}{D} \right) \times D \right)}{2} - \Lambda - c \right) \times g_{kl\mu\nu} \\
&\equiv d \times g_{kl\mu\nu}
\end{aligned}
\tag{183}
$$

Definition 2.73 (The n-index D dimensional d $_{\mathbf{kl}\mu\nu\,\ldots}$). *The n-index D dimensional* $d_{kl\mu\nu\,\ldots}$ *is defined* *as:*

$$
\begin{aligned}
d_{kl\mu\nu\ldots} &\equiv \left(\frac{\left(\left(\frac{R}{D}\right) \times D\right)}{2} - b \right) \times g_{kl\mu\nu\ldots} \\
&\equiv \left(\frac{\left(\left(\frac{R}{D}\right) \times D\right)}{2} - \Lambda - c \right) \times g_{kl\mu\nu\ldots} \\
&\equiv d \times g_{kl\mu\nu\ldots}
\end{aligned}
\tag{184}
$$

Table 4 provides an overview of the general definition of the relationships between the four basic (Barukčić, 2016a,b) fields of nature under conditions of the general theory of relativity.

		Curvature		
		YES	NO	
Momentum	YES	$a_{\mu\nu}$	$b_{\mu\nu}$	$E_{\mu\nu}$
	NO	$c_{\mu\nu}$	$d_{\mu\nu}$	$\underline{E}_{\mu\nu}$
		$G_{\mu\nu}$	$\underline{G}_{\mu\nu}$	$R_{\mu\nu}$

Table 4. Einstein field equations and the four basic fields of nature

2.9. *Axioms*

Whether science needs new and obviously generally valid statements (axioms) which are able to assure the truth of theorems proved from them may remain an unanswered question. In order to be accepted, a new axiom candidate (see Easwaran, 2008) should be at least as simple as possible and logically consistent to enable advances in our knowledge of nature. The importance of axioms is particularly emphasized by Albert Einstein. "**Die wahrhaft großen Fortschritte der Naturerkenntnis sind auf einem der Induktion fast diametral entgegengesetzten Wege entstanden**." (see Einstein, 1919, p. 17). In general, lex identitatis, lex contradictionis and lex negationis have the potential to denote the most simple, the most general and the most far-reaching axioms of science, the foundation of our today's and of our future scientific inquiry.

2.9.1. Principium identitatis (Axiom I)

Principium identitatis or **lex identitatis** or axiom I, is closely related to central problems of metaphysics, epistemology and of science as such. It turns out that it is more than rightful to assume that

$$+1 \equiv +1 \tag{185}$$

is true, otherwise there is every good reason to suppose that nothing can be discovered at all.

Identity as the epitome of a self-identical is at the same time different from difference, identity is free from difference, identity is at the same time the other of itself, identity is not difference. Identity is in its very own nature different, it is in its own self the opposite of itself (symmetry). It is equally

$$-1 \equiv -1 \tag{186}$$

In general, +1 and -1 are distinguished, however these distinct are related to one and the same 1. Identity as a vanishing of otherness, therefore, is this distinguishedness in one relation. It is

$$0 \equiv +1 - 1 \equiv 0 \times 1 \equiv 0 \tag{187}$$

Identity, as the unity of something and its own other is in its own self a separation from difference, and as a moment of separation might pass over into an equivalence relation which itself is reflexive, symmetric and transitive. Nonetheless, backed by thousands of years of often bitter human experience, the scientific development has taught us all that human knowledge is relative too. Even if experiments and other suitable proofs are of help to encourage us more and more in our belief of the correctness of a theory, it is difficult to prove the correctness of a theorem or of a theory et cetera once and for all. The challenge for all the science is the need to comply with Einstein's position: **"Niemals aber kann die Wahrheit einer Theorie erwiesen werden. Denn niemals weiß man, daß auch in Zukunft eine Erfahrung bekannt werden wird, die Ihren Folgerungen widerspricht..."** (Einstein, 1919). Albert Einstein's position translated into English: 'But the truth of a theory can never be proven. For one never knows if future experience will contradict its conclusion; and furthermore, there are always other conceptual systems imaginable which might coordinate the very same facts.'Our human experience tells us that everything in life is more or less transitory, and that nothing lasts. As a result of our knowledge and experience, several scientific theories have a glorious past to look back on, but all the glory of such scientific theories might remain in the past if scientist don't continue to innovate. In a word, theories can be refuted by time.

> "No amount of experimentation can ever prove me right;
> **a single experiment can prove me wrong**."
>
> (Albert Einstein according to: Robertson, 1998, p. 114)

In the light of the foregoing, it is clear that appropriate axioms and conclusions derived from the same are a main logical foundation of any 'theory'.

> "**Grundgesetz (Axiome)** und **Folgerungen** zusammen bilden das was man **eine 'Theorie'** nennt. "
>
> (Einstein, 1919)

However, another point is worth being considered again. One single experiment can be enough to refute a whole theory. Albert Einstein's (1879-1955) message translated into English as: *Basic law (axioms) and conclusions together form what is called a 'theory'* has still to get round. However, an axiom as a free creation of the human mind which precedes all science should be like all other axioms, as simple as possible and as self-evident as possible. Historically, the earliest documented use of **the law of identity** can be found in Plato's dialogue Theaetetus (185a) as "... each of the two is different from the other and the same as itself "[6] . However, Aristotle (384–322 B.C.E.), Plato's pupil and equally one of the greatest philosophers of all time, elaborated on the law of identity too. In Metaphysica, Aristotle wrote:

> "... all things ... have some unity and identity. "
>
> (see Aristotle, of Stageira (384-322 B.C.E), 1908, Metaphysica, Chapter IV, 999a, 25-30, p. 66)

In Prior Analytics, [7] , [8] Aristotle, a tutor Alexander, the thirteen-year-old son of Philip, the king of Macedon, is writing: "When A applies to the whole of B and of C, and is other predicated of nothing

[6] Plato's dialogue Theaetetus (185a), p. 104.
[7] Aristotle, Prior Analytics, Book II, Part 22, 68a
[8] Kenneth T. Barnes. Aristotle on Identity and Its Problems. Phronesis. Vol. 22, No. 1 (1977), pp. 48-62 (15 pages)

else, and B also applies to all C, A and B must be convertible. For since A is stated only of B and C, and B is predicated both of itself and of C, it is evident that B will also be stated of all subjects of which A is stated, except A itself. "[9], [10] For the sake of completeness, it should be noted at the outset that Aristotle himself preferred **the law of contradiction** and **the law of excluded middle** as examples of fundamental axioms. Nonetheless, it is worth noting that **lex identitatis** is an axiom too, which possess the potential to serve as the most basic and equally the most simple axiom of science but has been treated by Aristotle in an inadequate manner without having any clear and determined meaning for Aristotle himself. Nonetheless, something which is really just itself is equally different from everything else. In point of fact, is such an equivalence (Degen, 1741) which everything has to itself inherent or must the same be constructed by human mind and consciousness. Can and how can something be **identical with itself** (Förster and Melamed, 2012, Hegel, Georg Wilhelm Friedrich, 1812a, Koch, 1999, Newstadt, 2015) and in the same respect different from itself. An increasingly popular view on identity is the one advocated by Gottfried Wilhelm Leibniz (1646-1716):

"Chaque chose est ce qu'elle est. Et dans autant d'exemples qu'on voudra
A est A,
B est B. "
(Leibniz, 1765, p. 327)

or **A = A**, **B = B** or **+1 = +1**. In other words, a thing is what it is (Leibniz, 1765, p. 327). Leibniz' **principium identitatis indiscernibilium** (p.i.i.), the principle of the indistinguishable, occupies a central position in Leibniz' logic and metaphysics and was formulated by Leibniz himself in different ways in different passages (1663, 1686, 1704, 1715/16). All in all, Leibniz writes:

"C'est
le principe des indiscernables,
en vertu duquel
il ne saurait exister dans la nature deux êtres identiques.
...
Il n'y a point deux individus indiscernables. "
(see Leibniz, Gottfried Wilhelm, 1886, p. 45)

Exactly in complete compliance with Leibniz, Johann Gottlieb Fichte (1762 - 1814) elaborates on this subject as follows:

[9] Aristotle, Prior Analytics, Book II, Part 22, 68a, p. 511.
[10] Ivo Thomas. On a passage of Aristotle. Notre Dame J. Formal Logic 15(2): 347-348 (April 1974). DOI: 10.1305/ndjfl/1093891315

“Each thing is what it is ;
it has those realities which are posited when it is posited,
(A = A.) ”
(Fichte, 1889)

Hegel preferred to reformulate an own version of Leibnitz principium identitatis indiscernibilium in his own way by writing that “All things are different, or: there are no two things like each other. ”(see Hegel, Georg Wilhelm Friedrich, 1991, p. 422). Much of the debate about identity is still a matter of controversy. This issue has attracted the attention of many authors and has been discussed by Hegel too. In this context, it is worth to consider Hegel’s radical position on identity.

“The other expression of the law of identity: A cannot at the same time be A and not-A, has a negative form; it is called
the law of contradiction. ”
(Hegel, Georg Wilhelm Friedrich, 1991, p. 416)

We may, usefully (see Barukčić, 2019a), state Russell’s position with respect to the identity law as mentioned in his book ‘The problems of philosophy ’ (see Russell, 1912). In particular, according to Russell,

“...principles have been singled out by tradition under the name of ‘Laws of Thought.’ They are as follows:
(1) **The law of identity**: ‘Whatever is,is.
(2)**The law of contradiction**: ‘Nothing can both be and not be.’
(3) The law of excluded middle: ‘Everything must either be or not be.’
These three laws are samples of self-evident logical principles, but are not really more fundamental or more self-evident than various other similar principles: for instance, the one we considered just now, which states that what follows from a true premise is true. The name ‘laws of thought’ is also misleading, for what is important is not the fact that we think in accordance with these laws, but the fact that **things behave in accordance with them**; ”

(see Russell, 1912, p. 113)

Russell’s critique, that we tend too much to focus only on the formal aspects of the ‘Laws of Thoughts’ with the consequence that “... we thing in accordance with these laws” (see Russell, 1912, p. 113) is justified. Judged solely in terms of this aspect, it is of course necessary to think in accordance with the ‘Laws of Thoughts’. But this is not the only aspect of the ‘Laws of Thoughts’. The other and may be much more important aspect of these ‘Laws of Thoughts’is the fact that quantum mechanical objects or that “... things behave in accordance with them” (see Russell, 1912, p. 113).

2.9.2. Principium contradictionis (Axiom II)

Principium contradictionis or **lex contradictionis** [11], [12], [13] or axiom II, the other of lex identitatis, the negative of lex identitatis, the opposite of lex identitatis, a complementary of lex identitatis, can be expressed mathematically as

$$+0 \equiv 0 \times 1 \equiv +1 \tag{188}$$

In addition to the above, from the point of view of mathematics, axiom II (equation 188) is equally the most simple mathematical expression and formulation of a contradiction. However, there is too much practical and theoretical evidence that a lot of 'secured'mathematical knowledge and rules differ too generously from real world processes, and the question may be asked whether mathematical truths can be treated as absolute truths at all. Many of the basic principle of today's mathematics allow every single author defining the real world events and processes et cetera in a way as everyone likes it for himself. Consequentially, a resulting dogmatic epistemological subjectivism and at the end agnosticism too, after all, is one of the reasons why we should rightly heed the following words of wisdom of Albert Einstein.

"I don't believe in mathematics."

(Albert Einstein cited according to Brian, 1996, p. 76)

In the long term, however, the above attitude of mathematics is not sustainable. History has taught us time and time again that objective reality has the potential to correct wrong human thinking slowly but surely, and many more than this. Objective reality has demonstrably corrected wrong human thinking again and again in the past.

Despite all the adversities, it is necessary and crucial to consider that a self-identical as the opposite of itself is no longer only self-identity but a difference of itself from itself within itself. In other words, in opposition, a self-identical is able to return into simple unity with itself with the consequence that even as a self-identical the same self-identical is inherently self-contradictory. A question of fundamental theoretical importance is, however, why should something be itself and at the same time

[11] Horn, Laurence R., "Contradiction", The Stanford Encyclopedia of Philosophy (Winter 2018 Edition), Edward N. Zalta (ed.), URL = https://plato.stanford.edu/archives/win2018/entries/contradiction/.

[12] Barukčić I. Aristotle's law of contradiction and Einstein's special theory of relativity. Journal of Drug Delivery and Therapeutics (JDDT). 15Mar.2019;9(2):125-43. https://jddtonline.info/index.php/jddt/article/view/2389

[13] Barukčić, Ilija. (2020, December 28). The contradiction is exsiting objectively and real (Version 1). Zenodo. https://doi.org/10.5281/zenodo.4396106

the other of itself, the opposite of itself, not itself? Is something like this even possible at all and if so, why and how? These and similar questions have occupied many thinkers, including Hegel.

> "Something is therefore
> alive only in so far as it contains contradiction within it,
> and moreover is this power to
> hold and endure the contradiction within it. "
>
> (see Hegel, Georg Wilhelm Friedrich, 1991, p. 440)

However, as directed against identity, contradiction itself is also at the same time a source of self-changes out of itself of a self-identical.

> "... contradiction
> is the root of all movement and vitality;
> it is only in so far as something has a contradiction within it
> that it moves, has an urge and activity. "
>
> (see Hegel, Georg Wilhelm Friedrich, 1991, p. 439)

The further advance of science will throw any contribution to scientific progress of each of us back into scientific insignificance, as long as principium contradictioni is not given enough and the right attention. **The contradiction [14] is existing objectively and real and is the heartbeat of every self-identical.** We have reason to be delighted by the fact that very different aspects of principium contradictionis have been examined since centuries from different angles by various authors. According to Aristotle, principium contradictionis applies to everything that is.

> "... the same ... cannot at the same time belong and not belong to the same
> ... in the same respect ... This, then, is
> the most certain of all principles "
>
> (see Aristotle, of Stageira (384-322 B.C.E), 1908, Metaph., IV, 3, 1005b, 16–22)

Principium contradictionis or axiom II has many facets. As long as we follow Leibniz in this regard, we should consider that "**Le principe de contradiction est en general ...** "(Leibniz, 1765, p. 327). Scientist inevitably have false beliefs and make mistakes. In order to prevent scientific results from falling into logical inconsistency or logical absurdity, it is necessary to posses among other the methodological possibility to start a reasoning with a (logical) contradiction too. However and in contrast to the way of reasoning with inconsistent premises as proposed by para-consistent (Carnielli and

[14] Barukčić, Ilija. (2020, December 28). The contradiction is existing objectively and real (Version 1). Zenodo. https://doi.org/10.5281/zenodo.4396106

Marcos, 2001, da Costa, 1974, 1958, Priest, 1998, Priest et al., 1989, Quesada, 1977) and other logic, in the absence of technical and other errors of reasoning, the contradiction itself need to be preserved. In other words, **from a contradiction does not anything follows but the contradiction itself** while the theoretical question is indeed justified "What is so Bad about Contradictions? " (Priest, 1998). Historically, **the principle of (deductive) explosion** (Carnielli and Marcos, 2001, Priest, 1998, Priest et al., 1989), coined by 12th-century French philosopher William of Soissons, demand us to accept that anything, including its own negation, can be proven or can be inferred from a contradiction. In short, according to **ex falso sequitur quodlibet**, a (logical) contradiction implies anything. Respecting the principle of explosion, the existence of a contradiction (or the existence of logical inconsistency) in a scientific theorem, rule et cetera is disastrous. However, the historical development of science shows that scientist inevitably revise the theories, false positions and claims are identified once and again, and we all make different kind of mistakes. In order to avert disproportionately great damage to science and to prevent reducing science into pure subjective belief, a negation of the principle of explosion is required. Nonetheless, a justified negation of the **ex contradictione quodlibet principle** (Carnielli and Marcos, 2001) does not imply the correctness of para consistent logic (Carnielli and Marcos, 2001, da Costa, 1974, 1958, Priest, 1998, Priest et al., 1989, Quesada, 1977) as such as advocated especially by the Peruvian philosopher Francisco Miró Quesada (Quesada, 1977) and other (Carnielli and Marcos, 2001, da Costa, 1974, 1958, Priest, 1998, Priest et al., 1989). In general, scientific theories appear to progress from lower and simpler to higher and more complex levels. However, high level theories cannot be taken for granted because high level theories are grounded on a lot of assumptions, definitions and other procedures and may rest upon too much erroneous stuff even if still not identified. Therefore, it should be considered to check at lower at simpler levels like with like.

2.9.3. Principium negationis (Axiom III)

Lex negationis or axiom III, is often mismatched with simple opposition. However, from the point of view of philosophy and other sciences, identity, contradiction, negation and similar notions are equally mathematical descriptions of the most simple laws of objective reality. What sort of natural process is negation at the end? Mathematically, we define principium negationis or lex negationis or axiom III as

$$\text{Negation}(0) \times 0 \equiv \neg(0) \times 0 \equiv +1 \tag{189}$$

where $\neg$ denotes (logical (Boole, 1854) or natural) negation (Ayer, 1952, Förster and Melamed, 2012, Hedwig, 1980, Heinemann, 1943, Horn, 1989, Koch, 1999, Kunen, 1987, Newstadt, 2015, Royce, 1917, Speranza and Horn, 2010, Wedin, 1990b). In this context, there is some evidence that

$$\text{Negation}(1) \times 1 \equiv \neg(1) \times 1 = 0 \tag{190}$$

Logically, it follows that

$$\text{Negation}(1) \equiv 0 \tag{191}$$

In the following we assume that axiom I is universal. Under this assumption, the following theorem follows inevitably.

Theorem 2.2 (Zero divided by zero)**.** *According to classical logic, it is*

$$\frac{0}{0} \equiv 1 \tag{192}$$

Proof by direct proof. The premise

$$1 \equiv 1 \tag{193}$$

is true. It follows that

$$\begin{aligned} 0 &\equiv 0 \\ &\equiv 0 \times 1 \end{aligned} \tag{194}$$

In the following, we rearrange the premise (see equation 189, p. 57). We obtain

$$0 \times (\text{Negation}(0) \ \times 0) \equiv 0 \tag{195}$$

Equation 195 changes slightly (see equation 190, p. 57). It is

$$(\text{Negation}(1) \ \times 1) \times (\text{Negation}(0) \ \times 0) \equiv 0 \tag{196}$$

Equation 196 demands that

$$(\text{Negation}(1)\ ) \times (\text{Negation}(0)\ ) \times 0 \equiv 0 \tag{197}$$

Equation 197 is logically possible (see equation 187, p. 51) only if

$$(\text{Negation}(1)\ ) \times (\text{Negation}(0)\ ) \equiv 1 \tag{198}$$

Whatever the meaning of Negation(1) or of Negation(0) might be, equation 198 demands that

$$\text{Negation}(0) \ \equiv \frac{1}{\text{Negation}(1)} \tag{199}$$

and that

$$\text{Negation}(1) \equiv \frac{1}{\text{Negation}(0)} \tag{200}$$

Equation 199 simplifies as (see equation 191, p. 57)

$$\begin{aligned} \text{Negation}(0) \ &\equiv \frac{+1}{\text{Negation}(1)} \\ &\equiv \frac{+1}{+0} \end{aligned} \tag{201}$$

It follows that

$$\neg(0) \times 0 \equiv \frac{1}{0} \times 0 \equiv \frac{0}{0} \equiv 1 \tag{202}$$

To bring it to the point. Classical logic, assumed as generally valid, demands that

$$\frac{0}{0} \equiv 1 \tag{203}$$

□

Concepts like identity, difference, negation, opposition et cetera engaged the attention of scholars at least over the last twenty-three centuries (see also Horn, 1989, Speranza and Horn, 2010). As long as we first and foremost follow Josiah Royce, negatio or negation "is one of the simplest and most fundamental relations known to the human mind. For the study of logic, no more important and fruitful relation is known." (see also Royce, 1917, p. 265) But, do we really know what, for sure, what negation is? Based on what we know about negation, Aristotle (see also Wedin, 1990a) has been one of the first to present a theory of negation, which can be found in discontinuous chunks in his works the Metaphysics, the Categories, De Interpretatione, and the Prior Analytics (see also Horn, 1989, p. 1). Negation (see also Newstadt, 2015) as a fundamental philosophical concept found its own very special melting point especially in Hegel's dialectic and is more than just a formal logical process or operation which converts true to false or false to true. Negation as such is a natural process too and equally '**an engine of changes of objective reality** " (see also Barukčić, 2019a). However, it remains an open question to establish a generally accepted link between this fundamental philosophical concept and an adequate counterpart in physics, mathematics and mathematical statistics et cetera. Especially the relationship between creation and conservation or creatio ex nihilio (see also Donnelly, 1970, Ehrhardt, 1950, Ford, 1983), determination and negation (see also Ayer, 1952, Hedwig, 1980, Heinemann, 1943, Kunen, 1987) has been discussed in science since ancient (see also Horn, 1989, Speranza and Horn, 2010) times too. Why and how does an event occur or why and how is an event created (creation), why and how does an event maintain its own existence over time (conservation)? The development of the notion of negation leads from Aristotle to Meister Eckhart (see also Eckhart, 1986) von Hochheim (1260-1328), commonly known as Meister Eckhart (see also Tsopurashvili, 2012) or Eckehart, to Spinoza (1632 – 1677), to Immanuel Kant (1724-1804) and finally to Georg Wilhelm Friedrich Hegel (1770-1831) and other authors too. One point is worth being noted, even if it does not come as a surprise, it was especially Benedict de Spinoza (1632 – 1677) as one of the philosophical founding fathers of the Age of Enlightenment who addressed the relationship between determination and negation in his lost letter of June 2, 1674 to his friend Jarig Jelles (see also Förster and Melamed, 2012) by the discovery of his fundamental insight that " **determinatio negatio est**" (see also Spinoza, 1674, p. 634). Hegel went even so far as to extended the slogan raised by Spinoza into to "Omnis determinatio est negatio" (see also Hegel, Georg Wilhelm Friedrich, 1812b, 2010, p. 87). Finally, it did not take too long, and the notion of negation entered the world of mathematics and mathematical logic at least with Boole's (see also Boole, 1854) publication in the year 1854. "Let us, for simplicity of conception, give to the symbol x the particular interpretation of men, then 1 - x will represent the class of 'not-men'." (see also Boole, 1854, p. 49). Finally, the philosophical notion negation found its own way into physics by the contributions of authors like Woldemar Voigt (see Voigt, 1887), George Francis FitzGerald (see FitzGerald, 1889), Hendrik Antoon Lorentz (see Lorentz, 1892, 1899), Joseph Larmor (see Larmor, 1897), Jules Henri Poincaré (see Poincaré, 1905) and Albert Einstein (see Einstein, 1905b) by contributions to the physical notion "Lorentz factor".

3. Results

3.1. Energy time and space

Theorem 3.1 (Space is determined by energy and a third)**.** *In general, space is determined by energy and a third as*

$$_{R}S_{t} \equiv +\left({}_{R}E_{t}\right) + \left({}_{R}g_{t} \times c^{2}\right) \tag{204}$$

Proof by direct proof. **If** the premise

$$\underbrace{+1 = +1}_{(Premise)} \tag{205}$$

is true, **then** the following conclusion

$$_{\mathrm{R}}S_{\mathrm{t}} \equiv +\left({}_{\mathrm{R}}E_{\mathrm{t}}\right) + \left({}_{\mathrm{R}}g_{\mathrm{t}} \times c^{2}\right) \tag{206}$$

is also true, again the absence of any technical errors presupposed. The premise

$$+1 \equiv +1 \tag{207}$$

is true. We multiply equation 207 through by $_{\mathrm{R}}\mathrm{U}_{\mathrm{t}}$. It is

$$_{\mathrm{R}}U_{\mathrm{t}} \equiv {}_{\mathrm{R}}U_{\mathrm{t}} \tag{208}$$

We add zero to equation 208. It is

$$_{\mathrm{R}}U_{\mathrm{t}} + 0 \equiv {}_{\mathrm{R}}U_{\mathrm{t}} + 0 \tag{209}$$

Matter, denoted by $_{\mathrm{R}}\mathrm{M}_{\mathrm{t}}$ from the point of view of a stationary observer R, is one determining part of equation 209. Equation 209 changes to

$$_{\mathrm{R}}U_{\mathrm{t}} \equiv {}_{\mathrm{R}}U_{\mathrm{t}} - {}_{\mathrm{R}}M_{\mathrm{t}} + {}_{\mathrm{R}}M_{\mathrm{t}} \tag{210}$$

Following Einstein's path of thought, it is "... 'Materie'... **alles außer dem Gravitationsfeld** ..." (Einstein, 1916, p. 802/803) or all but matter is gravitational field. Equation 210 becomes in accordance with equation 133

$$_{\mathrm{R}}U_{\mathrm{t}} \equiv {}_{\mathrm{R}}g_{\mathrm{t}} + {}_{\mathrm{R}}M_{\mathrm{t}} \tag{211}$$

Multiplying equation 211 by the term $\left(\sqrt{1-\frac{v^{2}}{c^{2}}}\right)$ it is

$$\left({}_{\mathrm{R}}U_{\mathrm{t}} \times \sqrt{1-\frac{v^{2}}{c^{2}}}\right) \equiv \left({}_{\mathrm{R}}g_{\mathrm{t}} \times \sqrt{1-\frac{v^{2}}{c^{2}}}\right) + \left({}_{\mathrm{R}}M_{\mathrm{t}} \times \sqrt{1-\frac{v^{2}}{c^{2}}}\right) \tag{212}$$

where v is the relative velocity between the co-moving observer 0 and the stationary observer R, c is the speed of the light in vacuum. Equation 212 simplifies according to equation 135, equation 136 and equation 137 as

$$_{0}U_{\mathrm{t}} \equiv {}_{0}g_{\mathrm{t}} + {}_{0}M_{\mathrm{t}} \tag{213}$$

Normalizing the relationship between matter and gravitational field from the point of view of a stationary observer R, equation 211 becomes

$$\frac{_{R}g_{t}}{_{R}U_{t}} + \frac{_{R}M_{t}}{_{R}U_{t}} \equiv \frac{_{R}U_{t}}{_{R}U_{t}} \equiv +1 \tag{214}$$

Equation 214 demands that

$$_{R}g_{t} \equiv {_{R}U_{t}} \times \left(1 - \frac{_{R}M_{t}}{_{R}U_{t}}\right) \tag{215}$$

and equally that

$${_{R}g_{t}}^{2} \equiv {_{R}U_{t}}^{2} \times \left(1 - \frac{_{R}M_{t}}{_{R}U_{t}}\right)^{2} \tag{216}$$

Furthermore, equation 214 demands that

$$_{R}M_{t} \equiv {_{R}U_{t}} \times \left(1 - \frac{_{R}g_{t}}{_{R}U_{t}}\right) \tag{217}$$

and that

$${_{R}M_{t}}^{2} \equiv {_{R}U_{t}}^{2} \times \left(1 - \frac{_{R}g_{t}}{_{R}U_{t}}\right)^{2} \tag{218}$$

Normalizing the relationship between matter and gravitational field from the point of view of a co-moving observer 0, equation 213 becomes

$$\frac{_{0}g_{t}}{_{0}U_{t}} + \frac{_{0}m_{t}}{_{0}U_{t}} \equiv \frac{_{0}U_{t}}{_{0}U_{t}} \equiv +1 \tag{219}$$

Equation 219 demands that

$$_{0}g_{t} \equiv {_{0}U_{t}} \times \left(1 - \frac{_{0}m_{t}}{_{0}U_{t}}\right) \tag{220}$$

and equally that

$${_{0}g_{t}}^{2} \equiv {_{0}U_{t}}^{2} \times \left(1 - \frac{_{0}m_{t}}{_{0}U_{t}}\right)^{2} \tag{221}$$

Furthermore, equation 219 demands that

$$_{0}m_{t} \equiv {_{0}U_{t}} \times \left(1 - \frac{_{0}g_{t}}{_{0}U_{t}}\right) \tag{222}$$

and that

$${_{0}m_{t}}^{2} \equiv {_{0}U_{t}}^{2} \times \left(1 - \frac{_{0}g_{t}}{_{0}U_{t}}\right)^{2} \tag{223}$$

According to equation 212 it is

$$({_{0}g_{t}}) \equiv \left({_{R}g_{t}} \times \sqrt{1 - \frac{v^{2}}{c^{2}}}\right) \tag{224}$$

or

$$ {}_{0}g_{t}{}^{2} \equiv \left({}_{\mathrm{R}}g_{\mathrm{t}} \times \sqrt{1 - \frac{v^2}{c^2}} \right)^{2} \tag{225} $$

or

$$ \frac{{}_{0}g_{t}{}^{2}}{{}_{\mathrm{R}}g_{t}{}^{2}} \equiv \left(1 - \frac{v^2}{c^2} \right) \tag{226} $$

Multiplying equation 211 by c^2, it is

$$ \left({}_{\mathrm{R}}U_{\mathrm{t}} \times c^2 \right) \equiv \left({}_{\mathrm{R}}g_{\mathrm{t}} \times c^2 \right) + \left({}_{\mathrm{R}}M_{\mathrm{t}} \times c^2 \right) \tag{227} $$

According to equation 119, equation 227 becomes

$$ \left({}_{\mathrm{R}}U_{\mathrm{t}} \times c^2 \right) \equiv \left({}_{\mathrm{R}}g_{\mathrm{t}} \times c^2 \right) + \left({}_{\mathrm{R}}E_{\mathrm{t}} \right) \tag{228} $$

According to equation 133, equation 228 changes again. In general, it is

$$ \left({}_{\mathrm{R}}S_{\mathrm{t}} \right) \equiv + \left({}_{\mathrm{R}}E_{\mathrm{t}} \right) + \left({}_{\mathrm{R}}g_{\mathrm{t}} \times c^2 \right) \tag{229} $$

□

Energy ($_{R}E_{t}$) is a determining part of space ($_{R}S_{t}$), however something else, derived as $\left(_{R}g_{t} \times c^2 \right)$, which is different from the gravitational field ($_{R}g_{t}$), too. It is necessary to highlight at least one of the main differences between special theory of relativity and general theory of relativity. Special theory of relativity works more with the gravitational field, while general relativity works with the gravitational potential, both are deeply related but not completely identical.

3.2. The relativistic Schrödinger equation

Theorem 3.2 (The relativistic Schrödinger equation)**.** *The relativistic Schrödinger equation is given by the relationship*

$$ \left(_{P}E_{t} \times \Psi \right) + \left(_{K}E_{t} \times \Psi \right) \equiv H \times \Psi \tag{230} $$

where H is the Hamiltonian operator, Ψ is the wave function, $_{P}E_{t}$ is the relativistic potential energy and $_{K}E_{t}$ is the relativistic kinetic energy.

Proof by direct proof. **If** the premise

$$ \underbrace{+1 = +1}_{(Premise)} \tag{231} $$

is true, **then** the following conclusion

$$ \left({}_{\mathrm{P}}E_{\mathrm{t}} \times \Psi \right) + \left({}_{\mathrm{K}}E_{\mathrm{t}} \times \Psi \right) \equiv H \times \Psi \tag{232} $$

is also true, again the absence of any technical errors presupposed. The premise

$$+1 \equiv +1 \tag{233}$$

is true. We multiply equation 233 through by ${}_{\mathrm{R}}\mathrm{M}_{\mathrm{t}}$. It is

$${}_{\mathrm{R}}M_{\mathrm{t}} \equiv {}_{\mathrm{R}}M_{\mathrm{t}} \tag{234}$$

Multiplying equation 234 by the term $\left(\sqrt{1-\frac{v^2}{c^2}}\right)$ it is

$${}_{\mathrm{R}}M_{\mathrm{t}} \times \left(\sqrt{1-\frac{v^2}{c^2}}\right) \equiv {}_{\mathrm{R}}M_{\mathrm{t}} \times \left(\sqrt{1-\frac{v^2}{c^2}}\right) \tag{235}$$

Equation 235 becomes

$${}_{0}m_{\mathrm{t}} \equiv {}_{\mathrm{R}}M_{\mathrm{t}} \times \left(\sqrt{1-\frac{v^2}{c^2}}\right) \tag{236}$$

Equation 236 changes to

$${}_{0}m_{\mathrm{t}}{}^{2} \equiv {}_{\mathrm{R}}M_{\mathrm{t}}{}^{2} \times \left(\sqrt{1-\frac{v^2}{c^2}}\right)^{2} \tag{237}$$

and to

$$\frac{{}_{0}m_{\mathrm{t}}{}^{2}}{{}_{\mathrm{R}}M_{\mathrm{t}}{}^{2}} \equiv \left(1-\frac{v^2}{c^2}\right) \tag{238}$$

Furthermore, it is

$$\frac{c^2 \times c^2 \times {}_{0}m_{\mathrm{t}}{}^{2}}{c^2 \times c^2 \times {}_{\mathrm{R}}M_{\mathrm{t}}{}^{2}} \equiv \left(1-\frac{v^2}{c^2}\right) \tag{239}$$

or

$$\frac{{}_{0}E_{\mathrm{t}}{}^{2}}{{}_{\mathrm{R}}E_{\mathrm{t}}{}^{2}} \equiv \left(1-\frac{v^2}{c^2}\right) \tag{240}$$

Equation 240 changes to the normalized relativistic energy momentum relation as

$$\left(\frac{{}_{0}E_{\mathrm{t}}{}^{2}}{{}_{\mathrm{R}}E_{\mathrm{t}}{}^{2}}\right) + \left(\frac{v^2}{c^2}\right) \equiv +1 \tag{241}$$

Rearranging equation 241, it is

$$\left(\frac{{}_{0}E_{\mathrm{t}}{}^{2}}{{}_{\mathrm{R}}E_{\mathrm{t}}{}^{2}}\right) + \left(\frac{{}_{\mathrm{R}}M_{\mathrm{t}} \times v^2}{{}_{\mathrm{R}}M_{\mathrm{t}} \times c^2}\right) \equiv +1 \tag{242}$$

We define the relativistic kinetic(Barukčić, 2013, 2016b) energy, denoted by ${}_{\mathrm{K}}\mathrm{E}_{\mathrm{t}}$, something similar to Leibniz' vis viva (Barukčić, 2016b, Leibniz, 1686, 1695), as

$${}_{\mathrm{K}}E_{\mathrm{t}} \equiv {}_{\mathrm{R}}M_{\mathrm{t}} \times v^2 \tag{243}$$

Based on equation 243, the energy of an electromagnetic wave, ${}_{\mathrm{W}}\mathrm{E}_{\mathrm{t}}$, follows as

$$ {}_{\mathrm{W}}E_{\mathrm{t}}{}^{2} \equiv ({}_{\mathrm{K}}E_{\mathrm{t}}) \times ({}_{\mathrm{R}}E_{\mathrm{t}}) \tag{244} $$

Equation 242 becomes

$$ \left(\frac{{}_{0}E_{\mathrm{t}}{}^{2}}{{}_{\mathrm{R}}E_{\mathrm{t}}{}^{2}}\right) + \left(\frac{{}_{\mathrm{K}}E_{\mathrm{t}}}{{}_{\mathrm{R}}E_{\mathrm{t}}}\right) \equiv +1 \tag{245} $$

Einstein himself is demanding the following “Jeglicher Energie E kommt also im Gravitationsfelde eine Energie der Lage zu, die ebenso groß ist, wie die Energie der Lage einer ‘ponderablen’ Masse von der Größe E/c²”(Einstein, 1908). Translated into English: ‘Thus, to each energy E in the gravitational field, there corresponds an energy of position that equals the potential energy of a ‘ponderable’ mass of magnitude E/c²’. Following Einstein, we define the relativistic potential(Barukčić, 2013, 2016b) energy, ${}_{\mathrm{P}}\mathrm{E}_{\mathrm{t}}$, as

$$ {}_{\mathrm{P}}E_{\mathrm{t}} \equiv \left(\frac{{}_{0}E_{\mathrm{t}}{}^{2}}{{}_{\mathrm{R}}E_{\mathrm{t}}}\right) \equiv \left(\frac{{}_{0}E_{\mathrm{t}}}{{}_{\mathrm{R}}E_{\mathrm{t}}}\right) \times {}_{0}E_{\mathrm{t}} \equiv \left(\sqrt{1-\frac{v^{2}}{c^{2}}}\right) \times {}_{0}E_{\mathrm{t}} \tag{246} $$

Equation 245 becomes

$$ \left(\frac{{}_{\mathrm{P}}E_{\mathrm{t}}}{{}_{\mathrm{R}}E_{\mathrm{t}}}\right) + \left(\frac{{}_{\mathrm{K}}E_{\mathrm{t}}}{{}_{\mathrm{R}}E_{\mathrm{t}}}\right) \equiv +1 \tag{247} $$

Multiplying equation 247 by the term $H \times \Psi$ where H is the Hamiltonian and Ψ is the wave function it is

$$ \left(\frac{{}_{\mathrm{P}}E_{\mathrm{t}} \times H \times \Psi}{{}_{\mathrm{R}}E_{\mathrm{t}}}\right) + \left(\frac{{}_{\mathrm{K}}E_{\mathrm{t}} \times H \times \Psi}{{}_{\mathrm{R}}E_{\mathrm{t}}}\right) \equiv H \times \Psi \tag{248} $$

In quantum mechanics, the Hamiltonian of a certain system, denoted by H, is an operator corresponding to the total energy of a certain system. In the special theory of relativity, the total energy of a certain system is given by ${}_{\mathrm{R}}\mathrm{E}_{\mathrm{t}}$. However, the total energy of a certain system is the total energy of a certain system, there is no more energy left, independently of the notions and the mathematical framework used to describe the same system, both are identical. In the following, we assume that H = ${}_{\mathrm{R}}\mathrm{E}_{\mathrm{t}}$. Equation 248 changes a bit. The relativistic Schrödinger equation is given by the relationship

$$ ({}_{\mathrm{P}}E_{\mathrm{t}} \times \Psi) + ({}_{\mathrm{K}}E_{\mathrm{t}} \times \Psi) \equiv H \times \Psi \tag{249} $$

In general, based on equation 247 we must accept too, that

$$ ({}_{\mathrm{P}}E_{\mathrm{t}}) + ({}_{\mathrm{R}}E_{\mathrm{t}}) \equiv {}_{\mathrm{R}}E_{\mathrm{t}} \tag{250} $$

Nonetheless, equation 241 can be rearranged from another point of view. It is

$$ \left(\frac{{}_{0}E_{\mathrm{t}}{}^{2}}{{}_{\mathrm{R}}E_{\mathrm{t}}{}^{2}}\right) + \left(\frac{{}_{\mathrm{R}}M_{\mathrm{t}}{}^{2} \times v^{2} \times c^{2}}{{}_{\mathrm{R}}M_{\mathrm{t}}{}^{2} \times c^{2} \times c^{2}}\right) \equiv +1 \tag{251} $$

The relativistic momentum ${}_{\mathrm{R}}\mathrm{p}_{\mathrm{t}}$ is defined as ${}_{\mathrm{R}}p_{\mathrm{t}}{}^{2} \equiv {}_{\mathrm{R}}M_{\mathrm{t}}{}^{2} \times v^{2}$. Equation 251 simplifies as

$$ \left(\frac{{}_{0}E_{\mathrm{t}}{}^{2}}{{}_{\mathrm{R}}E_{\mathrm{t}}{}^{2}}\right) + \left(\frac{{}_{\mathrm{R}}p_{\mathrm{t}}{}^{2} \times c^{2}}{{}_{\mathrm{R}}E_{\mathrm{t}}{}^{2}}\right) \equiv +1 \tag{252} $$

The energy of an electromagnetic wave, denoted as ${}_{\mathrm{W}}\mathrm{E}_{\mathrm{t}}$, is defined as ${}_{\mathrm{W}}E_{\mathrm{t}}{}^{2} \equiv {}_{\mathrm{R}}p_{\mathrm{t}}{}^{2} \times c^{2} \equiv h^{2} \times {}_{\mathrm{R}}f_{\mathrm{t}}{}^{2}$ where h is Planck's constant and ${}_{\mathrm{R}}\mathrm{f}_{\mathrm{t}}$ is the frequency as determined by a stationary observer. Equation 252 changes to the particle-wave duality (Barukčić, 2013) relationship, as

$$\left(\frac{{}_{0}E_{\mathrm{t}}{}^{2}}{{}_{\mathrm{R}}E_{\mathrm{t}}{}^{2}}\right)+\left(\frac{{}_{\mathrm{W}}E_{\mathrm{t}}{}^{2}}{{}_{\mathrm{R}}E_{\mathrm{t}}{}^{2}}\right)\equiv +1 \tag{253}$$

Meanwhile we must once again repeat the following point. It is

$$\left(\frac{v^{2}}{c^{2}}\right)\equiv\left(\frac{{}_{\mathrm{R}}M_{\mathrm{t}}{}^{2}\times v^{2}\times c^{2}}{{}_{\mathrm{R}}M_{\mathrm{t}}{}^{2}\times c^{2}\times c^{2}}\right)\equiv\left(\frac{{}_{\mathrm{R}}p_{\mathrm{t}}{}^{2}\times c^{2}}{{}_{\mathrm{R}}E_{\mathrm{t}}{}^{2}}\right)\equiv\left(\frac{{}_{\mathrm{W}}E_{\mathrm{t}}{}^{2}}{{}_{\mathrm{R}}E_{\mathrm{t}}{}^{2}}\right) \tag{254}$$

Multiplying equation 253 by the term $H \times \Psi$ where H is the Hamiltonian and Ψ is the wave function it is

$$\left(\frac{{}_{0}E_{\mathrm{t}}{}^{2}}{{}_{\mathrm{R}}E_{\mathrm{t}}{}^{2}}\right)\times H\times\Psi+\left(\frac{{}_{\mathrm{W}}E_{\mathrm{t}}{}^{2}}{{}_{\mathrm{R}}E_{\mathrm{t}}{}^{2}}\right)\times H\times\Psi\equiv H\times\Psi \tag{255}$$

Under conditions where $H \equiv {}_{\mathrm{R}}E_{\mathrm{t}}$, the relativistic Schrödinger equation follows as

$$\left(\frac{{}_{0}E_{\mathrm{t}}{}^{2}\times\Psi}{{}_{\mathrm{R}}E_{\mathrm{t}}}\right)+\left(\frac{{}_{\mathrm{W}}E_{\mathrm{t}}{}^{2}\times\Psi}{{}_{\mathrm{R}}E_{\mathrm{t}}}\right)\equiv H\times\Psi \tag{256}$$

□

3.3. *The normalized relativistic time relation*

Theorem 3.3 (The normalized relativistic time relation)**.** *The normalized relativistic time relation is given by the equation*

$$\left(\frac{{}_{0}t_{t}{}^{2}}{{}_{R}t_{t}{}^{2}}\right)+\left(\frac{{}_{W}t_{t}{}^{2}}{{}_{R}t_{t}{}^{2}}\right)\equiv +1 \tag{257}$$

Proof by direct proof. **If** the premise

$$\underbrace{+1=+1}_{(Premise)} \tag{258}$$

is true, **then** the following conclusion

$$\left(\frac{{}_{0}t_{\mathrm{t}}{}^{2}}{{}_{\mathrm{R}}t_{\mathrm{t}}{}^{2}}\right)+\left(\frac{{}_{\mathrm{W}}t_{\mathrm{t}}{}^{2}}{{}_{\mathrm{R}}t_{\mathrm{t}}{}^{2}}\right)\equiv +1 \tag{259}$$

is also true, again the absence of any technical errors presupposed. The premise

$$+1\equiv +1 \tag{260}$$

is true. We multiply equation 260 through by the time as determined by a stationary observer R, ${}_{\mathrm{R}}\mathrm{t}_{\mathrm{t}}$. It is

$${}_{\mathrm{R}}t_{\mathrm{t}}\equiv {}_{\mathrm{R}}t_{\mathrm{t}} \tag{261}$$

Multiplying equation 261 by the term $\left(\sqrt{1-\frac{v^2}{c^2}}\right)$ it is

$$
{}_{\mathrm{R}}t_{\mathrm{t}} \times \left(\sqrt{1-\frac{v^2}{c^2}}\right) \equiv {}_{\mathrm{R}}t_{\mathrm{t}} \times \left(\sqrt{1-\frac{v^2}{c^2}}\right) \tag{262}
$$

According to Einstein(Einstein, 1905d, p. 904), equation 262 becomes

$$
{}_{0}t_{\mathrm{t}} \equiv {}_{\mathrm{R}}t_{\mathrm{t}} \times \left(\sqrt{1-\frac{v^2}{c^2}}\right) \tag{263}
$$

In the following, equation 263 changes to

$$
{}_{0}t_{\mathrm{t}}{}^2 \equiv {}_{\mathrm{R}}t_{\mathrm{t}}{}^2 \times \left(\sqrt{1-\frac{v^2}{c^2}}\right)^2 \tag{264}
$$

and to

$$
\frac{{}_{0}t_{\mathrm{t}}{}^2}{{}_{\mathrm{R}}t_{\mathrm{t}}{}^2} \equiv \left(1-\frac{v^2}{c^2}\right) \tag{265}
$$

Furthermore, it is

$$
\frac{c^2 \times c^2 \times {}_{0}t_{\mathrm{t}}{}^2}{c^2 \times c^2 \times {}_{\mathrm{R}}t_{\mathrm{t}}{}^2} \equiv \frac{{}_{0}t_{\mathrm{t}}{}^2}{c^2 \times c^2} \times \frac{c^2 \times c^2}{{}_{0}t_{\mathrm{t}}{}^2} \equiv \left(1-\frac{v^2}{c^2}\right) \tag{266}
$$

or (see equation 226)

$$
\frac{{}_{0}g_{\mathrm{t}}{}^2}{{}_{\mathrm{R}}g_{\mathrm{t}}{}^2} \equiv \left(1-\frac{v^2}{c^2}\right) \tag{267}
$$

Equation 265 changes to

$$
\left(\frac{{}_{0}t_{\mathrm{t}}{}^2}{{}_{\mathrm{R}}t_{\mathrm{t}}{}^2}\right) + \left(\frac{v^2}{c^2}\right) \equiv +1 \tag{268}
$$

and euqally to

$$
\left(\frac{v^2}{c^2}\right) \equiv 1 - \left(\frac{{}_{0}t_{\mathrm{t}}{}^2}{{}_{\mathrm{R}}t_{\mathrm{t}}{}^2}\right) \tag{269}
$$

Rearranging equation 268 further, it is

$$
\left(\frac{{}_{0}t_{\mathrm{t}}{}^2}{{}_{\mathrm{R}}t_{\mathrm{t}}{}^2}\right) + \left(\frac{{}_{\mathrm{R}}g_{\mathrm{t}}{}^2 \times v^2 \times c^2}{{}_{\mathrm{R}}g_{\mathrm{t}}{}^2 \times c^2 \times c^2}\right) \equiv +1 \tag{270}
$$

It is necessary (see equation 123 and 131) to be considered that

$$
\left(\frac{v^2}{c^2}\right) \equiv \left(\frac{{}_{\mathrm{R}}g_{\mathrm{t}}{}^2 \times v^2 \times c^2}{{}_{\mathrm{R}}g_{\mathrm{t}}{}^2 \times c^2 \times c^2}\right) \equiv \left(\frac{{}_{\mathrm{Kred}}t_{\mathrm{t}}{}^2 \times c^2}{{}_{\mathrm{R}}t_{\mathrm{t}}{}^2}\right) \equiv \left(\frac{{}_{\mathrm{W}}t_{\mathrm{t}}{}^2}{{}_{\mathrm{R}}t_{\mathrm{t}}{}^2}\right) \tag{271}
$$

Equation 271 demands too, that,

$$
{}_{\mathrm{W}}t_{\mathrm{t}}{}^2 \equiv \left(\frac{v^2}{c^2}\right) \times {}_{\mathrm{R}}t_{\mathrm{t}}{}^2 \tag{272}
$$

As known, the mathematical identity ${}_{\mathrm{W}}t_{\mathrm{t}}$ is defined as ${}_{\mathrm{W}}t_{\mathrm{t}}{}^{2} \equiv {}_{\mathrm{R}}g_{\mathrm{t}}{}^{2} \times v^{2} \times c^{2}$ (see equation 123). Equation 270 simplifies further. In general, the normalized relativistic time relation is given by the equation

$$\left(\frac{{}_{0}t_{\mathrm{t}}{}^{2}}{{}_{\mathrm{R}}t_{\mathrm{t}}{}^{2}}\right) + \left(\frac{{}_{\mathrm{W}}t_{\mathrm{t}}{}^{2}}{{}_{\mathrm{R}}t_{\mathrm{t}}{}^{2}}\right) \equiv +1 \tag{273}$$

□

Theorem 3.4 (Time is exsisting objectively and real)**.** *Taking Kant(see Kant, 1770) for granted, "Time is not something objective and real, neither a substance, nor an accident, nor a relation." (see also (English) Kant, 1894, p. 61). However, in complete contrast to Kant, Einstein's special theory of relativity demand us to accept that time is existing objectively and real. In general, we must consider that*

$$1 - \frac{v^{2}}{c^{2}} \equiv \frac{{}_{0}E_{t}{}^{2}}{{}_{R}E_{t}{}^{2}} \equiv \frac{{}_{0}t_{t}{}^{2}}{{}_{R}t_{t}{}^{2}} \tag{274}$$

Proof by direct proof. **If** the premise

$$\underbrace{+1 = +1}_{(Premise)} \tag{275}$$

is true, **then** the following conclusion

$$1 - \frac{v^{2}}{c^{2}} \equiv \frac{{}_{0}E_{\mathrm{t}}{}^{2}}{{}_{\mathrm{R}}E_{\mathrm{t}}{}^{2}} \equiv \frac{{}_{0}t_{\mathrm{t}}{}^{2}}{{}_{\mathrm{R}}t_{\mathrm{t}}{}^{2}} \tag{276}$$

is also true, again the absence of any technical errors presupposed. The premise

$$+1 \equiv +1 \tag{277}$$

is true. We multiply equation 277 through by the term $\left(1 - \frac{v^{2}}{c^{2}}\right)$, it is

$$1 - \frac{v^{2}}{c^{2}} \equiv 1 - \frac{v^{2}}{c^{2}} \tag{278}$$

Equation 278 changes to (see equation 240)

$$1 - \frac{v^{2}}{c^{2}} \equiv \frac{{}_{0}E_{\mathrm{t}}{}^{2}}{{}_{\mathrm{R}}E_{\mathrm{t}}{}^{2}} \tag{279}$$

Equation 279 changes (see equation 265) to

$$1 - \frac{v^{2}}{c^{2}} \equiv \frac{{}_{0}E_{\mathrm{t}}{}^{2}}{{}_{\mathrm{R}}E_{\mathrm{t}}{}^{2}} \equiv \frac{{}_{0}t_{\mathrm{t}}{}^{2}}{{}_{\mathrm{R}}t_{\mathrm{t}}{}^{2}} \tag{280}$$

□

Time itself is not depending on human mind and consciousness but existing objectively and real. The behaviour of time is linked and determined by the behaviour of energy (see equation 280) and/or vice versa, and not to human mind and consciousness. However, time itself is not energy, it is the other of energy, it is the complementary of energy, it is the opposite of energy. And what is most important, time itself has another form of existence than energy/matter, time is non-matter. Special relativity and equation 280 forces us to accept the objective existence of a non-material world, however the same might be organized by nature. Theoretically it would be conceivable that energy might pass over into time and time into energy. Space is this interaction between energy and time, or at least appears to be, the unity and the struggle of energy and time.

Theorem 3.5 (The First Basic Law of Special Relativity)**.** *The first(Barukčić, 2016b) basic law of special relativity is given by*

$$ {}_0E_t \times {}_Rt_t \equiv {}_0t_t \times {}_RE_t \tag{281} $$

Proof by direct proof. **If** the premise

$$ \underbrace{+1 = +1}_{(Premise)} \tag{282} $$

is true, **then** the following conclusion

$$ {}_0E_{\mathrm{t}} \times {}_{\mathrm{R}}t_{\mathrm{t}} \equiv {}_0t_{\mathrm{t}} \times {}_{\mathrm{R}}E_{\mathrm{t}} \tag{283} $$

is also true, again the absence of any technical errors presupposed. The premise

$$ +1 \equiv +1 \tag{284} $$

is true. We multiply equation 284 through by the term $\left(1 - \frac{v^2}{c^2}\right)$, it is

$$ 1 - \frac{v^2}{c^2} \equiv 1 - \frac{v^2}{c^2} \tag{285} $$

Equation 285 changes to (see equation 240)

$$ \frac{{}_0E_{\mathrm{t}}{}^2}{{}_{\mathrm{R}}E_{\mathrm{t}}{}^2} \equiv 1 - \frac{v^2}{c^2} \tag{286} $$

Equation 286 changes (see equation 265) to

$$ \frac{{}_0E_{\mathrm{t}}{}^2}{{}_{\mathrm{R}}E_{\mathrm{t}}{}^2} \equiv \frac{{}_0t_{\mathrm{t}}{}^2}{{}_{\mathrm{R}}t_{\mathrm{t}}{}^2} \tag{287} $$

Rearranging equation 287 it is

$$ {}_0E_{\mathrm{t}}{}^2 \times {}_{\mathrm{R}}t_{\mathrm{t}}{}^2 \equiv {}_0t_{\mathrm{t}}{}^2 \times {}_{\mathrm{R}}E_{\mathrm{t}}{}^2 \tag{288} $$

Equation 288 can be simplified. The first basic law of special relativity follows, as

$$ {}_0E_{\mathrm{t}} \times {}_{\mathrm{R}}t_{\mathrm{t}} \equiv {}_0t_{\mathrm{t}} \times {}_{\mathrm{R}}E_{\mathrm{t}} \tag{289} $$

□

The second(Barukčić, 2016b) basic law can be derived similarly.

Theorem 3.6 (Energy is determined by time)**.** *Energy is determined by time as given by the equation*

$$
{}_{R}E_{t}{}^{2} \equiv \frac{{}_{0}E_{t}{}^{2}}{{}_{0}t_{t}{}^{2}} \times {}_{R}t_{t}{}^{2} \tag{290}
$$

Proof by direct proof. According to equation 287 it is

$$
\frac{{}_{0}E_{t}{}^{2}}{{}_{R}E_{t}{}^{2}} \equiv \frac{{}_{0}t_{t}{}^{2}}{{}_{R}t_{t}{}^{2}} \tag{291}
$$

Rearranging equation 291, it is

$$
{}_{R}E_{t}{}^{2} \equiv \frac{{}_{0}E_{t}{}^{2}}{{}_{0}t_{t}{}^{2}} \times {}_{R}t_{t}{}^{2} \tag{292}
$$

□

Theorem 3.7 (Time is determined by energy)**.** *Time is determined by energy as indicated by the equation*

$$
{}_{R}t_{t}{}^{2} \equiv \frac{{}_{0}t_{t}{}^{2}}{{}_{0}E_{t}{}^{2}} \times {}_{R}E_{t}{}^{2} \tag{293}
$$

Proof by direct proof. According to equation 287 it is

$$
\frac{{}_{0}E_{t}{}^{2}}{{}_{R}E_{t}{}^{2}} \equiv \frac{{}_{0}t_{t}{}^{2}}{{}_{R}t_{t}{}^{2}} \tag{294}
$$

Rearranging equation 294, it is

$$
{}_{R}t_{t}{}^{2} \equiv \frac{{}_{0}t_{t}{}^{2}}{{}_{0}E_{t}{}^{2}} \times {}_{R}E_{t}{}^{2} \tag{295}
$$

□

Theorem 3.8 (Space as the unity and the struggle between energy and time)**.**

$$
\left({}_{R}E_{t}{}^{2}\right) + \left({}_{R}t_{t}{}^{2}\right) \equiv \left(\frac{{}_{0}E_{t}{}^{2}}{{}_{0}t_{t}{}^{2}} \times {}_{R}t_{t}{}^{2}\right) + \left(\frac{{}_{0}t_{t}{}^{2}}{{}_{0}E_{t}{}^{2}} \times {}_{R}E_{t}{}^{2}\right) \tag{296}
$$

Proof by direct proof. It is

$$
\left({}_{R}E_{t}{}^{2}\right) + \left({}_{R}t_{t}{}^{2}\right) \equiv \left({}_{R}E_{t}{}^{2}\right) + \left({}_{R}t_{t}{}^{2}\right) \tag{297}
$$

Equation 297 becomes (see equation 292)

$$
\left({}_{R}E_{t}{}^{2}\right) + \left({}_{R}t_{t}{}^{2}\right) \equiv \left(\frac{{}_{0}E_{t}{}^{2}}{{}_{0}t_{t}{}^{2}} \times {}_{R}t_{t}{}^{2}\right) + \left({}_{R}t_{t}{}^{2}\right) \tag{298}
$$

Equation 298 becomes (see equation 295)

$$
\left({}_{R}E_{t}{}^{2}\right) + \left({}_{R}t_{t}{}^{2}\right) \equiv \left(\frac{{}_{0}E_{t}{}^{2}}{{}_{0}t_{t}{}^{2}} \times {}_{R}t_{t}{}^{2}\right) + \left(\frac{{}_{0}t_{t}{}^{2}}{{}_{0}E_{t}{}^{2}} \times {}_{R}E_{t}{}^{2}\right) \tag{299}
$$

□

3.4. Energy, time and space I

Energy, time and space were addressed in the history of science, in the following mentioned only exemplarily, from Aristotle moving on to Newton, through to Leibniz, Kant and other over and over again with contradictory results. Even today's concepts of energy, time and space have not answered the fundamental question of what is the intrinsic nature of energy, time and space. So it is not surprising that Einstein himself seriously challenged the long period of dominance of Newton's conception of absolute space and of absolute time. However, in order to reply to the question of energy, time and space it might prove helpful to evaluate once again Einstein's understanding of view fundamental relationships of nature. And that is why, with the utmost admiration and joy, it is my great privilege to be able to refer to what Einstein himself wrote:

> "Wir unterscheiden im folgenden zwischen 'Gravitationsfeld'und 'Materie', in dem Sinne, daß
>
> **alles außer dem Gravitationsfeld** als '**Materie**'bezeichnet wird, also nicht nur
>
> die **'Materie'im üblichen Sinne**, sondern auch das **elektromagnetische Feld**. "
>
> (Einstein, 1916, p. 802/803)

Finally, Einstein provide us by simple words of wisdom with the necessary knowledge to be able to distinguish the one from its own other. Einstein's understanding of the relationship between energy/matter which includes the electromagnetic field too and the gravitational field itself translated into English: *In the following we make a distinction between 'gravitational field' and 'matter' in this way, that we denote* ***everything but the gravitational field*** *as* ***'matter'****, that is to say not only the* ***'matter' in the ordinary sense****, but also the* ***electromagnetic field*** *as well.* In this regard, Einstein's position under consideration can be clarified and demonstrated easily by using the following picture.

Gravitational field

M a t t e r

Everything (i.e. $_R U_t$) as the unity and the struggle between matter and gravitational field.

Pushed to its extreme, Einstein defines matter **ex negativo** as **everything but the gravitational field,** in perfect agreement with Aristotle's law of the excluded middle. In other words, there is no third between matter and gravitational field, a third is not given, **tertium non datur**.

Definition 3.1 (Matter and gravitational field). *Let $E({}_R M_t)$ denote the expectation value of matter, let $E({}_R g_t)$ denote the expectation value of gravitational field, let ${}_R E_t$ denote everything. In general, it is*

$$ {}_R E_t \equiv E({}_R M_t) + E({}_R g_t) \tag{300} $$

The relationship between matter and gravitational field can be normalized.

Definition 3.2 (Matter and gravitational field normalized). *Again, let $E({}_{R}M_{t})$ denote matter, let $E({}_{R}g_{t})$ denote the gravitational field, let ${}_{R}E_{t}$ denote everything. The relationship between matter and gravitational field is normalized as*

$$\frac{E({}_{R}M_{t})}{{}_{R}E_{t}} + \frac{E({}_{R}g_{t})}{{}_{R}E_{t}} \equiv +1 \tag{301}$$

Definition 3.3 (Energy, time and space). *In our understanding, the space itself, as the set of all sets, denoted by ${}_{R}S_{t}$, is determined as*

$${}_{R}S_{t} \equiv {}_{R}U_{t} \times c^{2} \equiv E({}_{R}E_{t}) + E({}_{R}t_{t}) \tag{302}$$

while $E({}_{R}E_{t})$ is the expectation value of energy and $E({}_{R}t_{t})$ is the expectation value of time.

The relationship between energy and time can be normalized.

Definition 3.4 (Energy and time normalized). *Again, let $E({}_{R}E_{t})$ denote the expectation value energy, let $E({}_{R}t_{t})$ denote the expectation value time, let ${}_{R}S_{t}$ denote space. The relationship between energy and time is normalized as*

$$\frac{E({}_{R}E_{t})}{{}_{R}S_{t}} + \frac{E({}_{R}t_{t})}{{}_{R}S_{t}} \equiv +1 \tag{303}$$

Definition 3.5 (Energy and matter). *Based on Einstein's theory of special(Einstein, 1905c,d, 1908, 1935) relativity, the relationship between energy $E({}_{R}E_{t})$ and matter $E({}_{R}M_{t})$ is given by*

$$E({}_{R}E_{t}) \equiv E({}_{R}M_{t}) \times c^{2} \equiv E({}_{0}M_{t} + {}_{\Delta}M_{t}) \times c^{2} \tag{304}$$

while $E({}_{R}E_{t})$ is the expectation value of energy, $E({}_{0}M_{t})$ is the expectation value of mass as determined by a co-moving observer, c is the speed of the light in vacuum.

However, matter itself as the other of the gravitational field is determined by the **electromagnetic field** and **'matter' in the ordinary sense**. The next figure might illustrate these basic relationships in more detail.

Electromagnetic field $b_{\mu\nu}$

$\boxed{\textbf{Ordinary matter } \mathbf{a}_{\mu\nu}}$

Energy tensor as the relationship between ordinary matter and electromagnetic field.

Again, Einstein himself elaborates on the fundamental relationship between matter in the narrower sense and the electromagnetic field in detail. "***Considered phenomenologically, this energy tensor is composed of that of the electromagnetic field and of matter in the narrower sense.*** " (see Einstein, 1923b, p. 93) or in greater detail:

“But our investigations ... have shown that in ... the tensor of energy ... is included not only the tensor of the energy of ponderable matter, but also that of the electromagnetic energy. ”

(see Einstein, 1923b, p. 87)

Vranceanu (see Vranceanu, 1936) himself and other too is elaborating on the same issue, too. In point of fact, the energy tensor T_{kl} is treated by Vranceanu as the sum of two tensors, one of which is due to the electromagnetic field ($\mathrm{b}_{\mu\nu}$).

“On peut aussi supposer que le tenseur d’énergie T_{kl} soit la somme de deux tenseurs dont un dû au champ électromagnétique . . . ” (see Vranceanu, 1936)

Vranceanu (see Vranceanu, 1936) and the stress-energy tensor.

Translated into English: ‘*One can also assume that the energy tensor T_{kl} be the sum of two tensors, one of which is due to the electromagnetic field.*’In the light of these quite extensive information and clarifications, the issue of energy, time and space can be taken one step further towards our goal, which is to work out the fundamental relationship between these notions.

Theorem 3.9 (Energy, time and space)**.** *In general, it is*

$$\frac{E\left({}_{R}X_{t}\right)}{c^{2}} \equiv E\left({}_{R}g_{t}\right) \tag{305}$$

Proof by direct proof. **If** the premise

$$\underbrace{+1=+1}_{(Premise)} \tag{306}$$

is true, **then** the following conclusion

$$+1 \equiv +1 \tag{307}$$

is also true, again the absence of any technical errors presupposed. The premise

$$+1 \equiv +1 \tag{308}$$

is true. We multiply equation 308 through by $E(_{\mathrm{R}}E_{\mathrm{t}})$ it is

$$E\left({}_{\mathrm{R}}E_{\mathrm{t}}\right) \equiv E\left({}_{\mathrm{R}}E_{\mathrm{t}}\right) \tag{309}$$

Adding $E({}_{\mathrm{R}}X_{\mathrm{t}})$, equation 309 becomes

$$E\left({}_{\mathrm{R}}E_{\mathrm{t}}\right)+E\left({}_{\mathrm{R}}X_{\mathrm{t}}\right)\equiv E\left({}_{\mathrm{R}}E_{\mathrm{t}}\right)+E\left({}_{\mathrm{R}}X_{\mathrm{t}}\right) \tag{310}$$

At this stage of the proof, the exact nature of the term $E({}_{\mathrm{R}}X_{\mathrm{t}})$ is completely unknown. However, we know for sure that the relationship $E\left({}_{\mathrm{R}}E_{\mathrm{t}}\right)+E\left({}_{\mathrm{R}}X_{\mathrm{t}}\right)\equiv\left({}_{\mathrm{R}}S_{\mathrm{t}}\right)$ is true. In other words, there is no third between E(${}_{\mathrm{R}}\mathrm{E}_{\mathrm{t}}$) and E(${}_{\mathrm{R}}\mathrm{X}_{\mathrm{t}}$), a third is not given, tertium non datur! Thus far, albeit clothed in other words, energy is for sure a constituting part of space, but something else too. Therefore, equation 310 changes slightly. In general, it is,

$$E\left({}_{\mathrm{R}}E_{\mathrm{t}}\right)+E\left({}_{\mathrm{R}}X_{\mathrm{t}}\right)\equiv\left({}_{\mathrm{R}}S_{\mathrm{t}}\right) \tag{311}$$

However, what could be the meaning of $E({}_{\mathrm{R}}X_{\mathrm{t}})$? Based on equation 302, equation 311 becomes

$$E\left({}_{\mathrm{R}}E_{\mathrm{t}}\right)+E\left({}_{\mathrm{R}}X_{\mathrm{t}}\right)\equiv\left({}_{\mathrm{R}}U_{\mathrm{t}}\right)\times c^{2} \tag{312}$$

Dividing equation 312 by c^2, it is

$$\left(\frac{E({}_{\mathrm{R}}E_{\mathrm{t}})}{c^{2}}\right)+\left(\frac{E({}_{\mathrm{R}}X_{\mathrm{t}})}{c^{2}}\right)\equiv\left({}_{\mathrm{R}}U_{\mathrm{t}}\right) \tag{313}$$

According to equation 300 it is ${}_{\mathrm{R}}U_{\mathrm{t}}\equiv E({}_{\mathrm{R}}M_{\mathrm{t}})+E({}_{\mathrm{R}}g_{\mathrm{t}})$. Equation 313 changes to

$$\left(\frac{E({}_{\mathrm{R}}E_{\mathrm{t}})}{c^{2}}\right)+\left(\frac{E({}_{\mathrm{R}}X_{\mathrm{t}})}{c^{2}}\right)\equiv E\left({}_{\mathrm{R}}M_{\mathrm{t}}\right)+E\left({}_{\mathrm{R}}g_{\mathrm{t}}\right) \tag{314}$$

Based on equation 304, it is $E({}_{\mathrm{R}}M_{\mathrm{t}})\equiv\dfrac{E({}_{\mathrm{R}}E_{\mathrm{t}})}{c^{2}}$. Equation 314 becomes

$$E\left({}_{\mathrm{R}}M_{\mathrm{t}}\right)+\left(\frac{E({}_{\mathrm{R}}X_{\mathrm{t}})}{c^{2}}\right)\equiv E\left({}_{\mathrm{R}}M_{\mathrm{t}}\right)+E\left({}_{\mathrm{R}}g_{\mathrm{t}}\right) \tag{315}$$

Even if we do not expressly know what $\mathrm{E}({}_{\mathrm{R}}\mathrm{X}_{\mathrm{t}})$ really is, what we know for sure is that $\mathrm{E}({}_{\mathrm{R}}\mathrm{X}_{\mathrm{t}})$ is determined by the expectation value of the gravitational field $\mathrm{E}({}_{\mathrm{R}}\mathrm{g}_{\mathrm{t}})$. In general, it is

$$\frac{E({}_{\mathrm{R}}X_{\mathrm{t}})}{c^{2}}\equiv E\left({}_{\mathrm{R}}g_{\mathrm{t}}\right) \tag{316}$$

□

Theorem 3.10 (Energy, time and space II)**.** *Energy, time and space are deeply interrelated. It is*

$${}_{R}E_{t}{}^{2}\equiv{}_{R}S_{t}{}^{2}\times\left(1-\frac{{}_{R}t_{t}}{{}_{R}S_{t}}\right)^{2} \tag{317}$$

Proof by direct proof. In our understanding it is

$${}_{\mathrm{R}}E_{\mathrm{t}}+{}_{\mathrm{R}}t_{\mathrm{t}}\equiv{}_{\mathrm{R}}S_{\mathrm{t}} \tag{318}$$

Normalizing equation 318, it is

$$\frac{_{R}E_{t}}{_{R}S_{t}} + \frac{_{R}t_{t}}{_{R}S_{t}} \equiv \frac{_{R}S_{t}}{_{R}S_{t}} \equiv +1 \tag{319}$$

Changing equation 319, it is

$$_{R}E_{t} \equiv {}_{R}S_{t} \times \left(1 - \frac{_{R}t_{t}}{_{R}S_{t}}\right) \tag{320}$$

and at the end

$$_{R}E_{t}{}^{2} \equiv {}_{R}S_{t}{}^{2} \times \left(1 - \frac{_{R}t_{t}}{_{R}S_{t}}\right)^{2} \tag{321}$$

□

Theorem 3.11 (Energy, time and space III)**.** *Energy, time and space are deeply interrelated. It is equally*

$$_{R}t_{t}{}^{2} \equiv {}_{R}S_{t}{}^{2} \times \left(1 - \frac{_{R}E_{t}}{_{R}S_{t}}\right)^{2} \tag{322}$$

Proof by direct proof. In our understanding it is

$$_{R}E_{t} + {}_{R}t_{t} \equiv {}_{R}S_{t} \tag{323}$$

Normalizing equation 323, it is

$$\frac{_{R}E_{t}}{_{R}S_{t}} + \frac{_{R}t_{t}}{_{R}S_{t}} \equiv \frac{_{R}S_{t}}{_{R}S_{t}} \equiv +1 \tag{324}$$

Changing equation 324, it is

$$_{R}t_{t} \equiv {}_{R}S_{t} \times \left(1 - \frac{_{R}E_{t}}{_{R}S_{t}}\right) \tag{325}$$

and at the end

$$_{R}t_{t}{}^{2} \equiv {}_{R}S_{t}{}^{2} \times \left(1 - \frac{_{R}E_{t}}{_{R}S_{t}}\right)^{2} \tag{326}$$

□

Theorem 3.12 (Energy, time and space IV)**.** *The relationship between energy, time and space finds its most obvious expression mathematically in the equation*

$$\left(_{R}E_{t}{}^{2}\right) + \left(_{R}t_{t}{}^{2}\right) \equiv \kappa \times {}_{R}S_{t}{}^{2} \tag{327}$$

Proof by direct proof. It is

$$\left(_{R}E_{t}{}^{2}\right) + \left(_{R}t_{t}{}^{2}\right) \equiv \left(_{R}E_{t}{}^{2}\right) + \left(_{R}t_{t}{}^{2}\right) \tag{328}$$

Equation 328 becomes (see equation 321)

$$\left(_{R}E_{t}{}^{2}\right) + \left(_{R}t_{t}{}^{2}\right) \equiv \left({}_{R}S_{t}{}^{2} \times \left(1 - \frac{_{R}t_{t}}{_{R}S_{t}}\right)^{2}\right) + \left(_{R}t_{t}{}^{2}\right) \tag{329}$$

Equation 329 becomes (see equation 326)

$$\left({}_{\mathrm{R}}E_{\mathrm{t}}{}^{2}\right)+\left({}_{\mathrm{R}}t_{\mathrm{t}}{}^{2}\right)\equiv\left({}_{\mathrm{R}}S_{\mathrm{t}}{}^{2}\times\left(1-\frac{{}_{\mathrm{R}}t_{\mathrm{t}}}{{}_{\mathrm{R}}S_{\mathrm{t}}}\right)^{2}\right)+\left({}_{\mathrm{R}}S_{\mathrm{t}}{}^{2}\times\left(1-\frac{{}_{\mathrm{R}}E_{\mathrm{t}}}{{}_{\mathrm{R}}S_{\mathrm{t}}}\right)^{2}\right)\tag{330}$$

Simplifying equation 330, it is

$$\left({}_{\mathrm{R}}E_{\mathrm{t}}{}^{2}\right)+\left({}_{\mathrm{R}}t_{\mathrm{t}}{}^{2}\right)\equiv\left(\left(1-\frac{{}_{\mathrm{R}}t_{\mathrm{t}}}{{}_{\mathrm{R}}S_{\mathrm{t}}}\right)^{2}+\left(1-\frac{{}_{\mathrm{R}}E_{\mathrm{t}}}{{}_{\mathrm{R}}S_{\mathrm{t}}}\right)^{2}\right)\times{}_{\mathrm{R}}S_{\mathrm{t}}{}^{2}\tag{331}$$

For better handling, we define mathematical identity

$$\kappa\equiv\left(\left(1-\frac{{}_{\mathrm{R}}t_{\mathrm{t}}}{{}_{\mathrm{R}}S_{\mathrm{t}}}\right)^{2}+\left(1-\frac{{}_{\mathrm{R}}E_{\mathrm{t}}}{{}_{\mathrm{R}}S_{\mathrm{t}}}\right)^{2}\right)\tag{332}$$

In general, the relation between energy and time changes to

$$\left({}_{\mathrm{R}}E_{\mathrm{t}}{}^{2}\right)+\left({}_{\mathrm{R}}t_{\mathrm{t}}{}^{2}\right)\equiv\kappa\times{}_{\mathrm{R}}S_{\mathrm{t}}{}^{2}\tag{333}$$

□

The following figure provides an overview over the basic relationships as given from the point of view of special theory of relativity.

Table 5. Energy, time and space

		Curvature		
		YES	NO	
Momentum	YES	$\mathbf{{}_{0}E_{t}{}^{2}}$	${}_{\mathrm{W}}\mathrm{E}_{\mathrm{t}}{}^{2}$	${}_{\mathrm{R}}\mathrm{E}_{\mathrm{t}}{}^{2}$
	NO	${}_{0}\mathrm{t}_{\mathrm{t}}{}^{2}$	${}_{\mathrm{W}}\mathrm{t}_{\mathrm{t}}{}^{2}$	${}_{\mathrm{R}}\mathrm{t}_{\mathrm{t}}{}^{2}$
		${}_{0}\mathrm{S}_{\mathrm{t}}{}^{2}$	${}_{0}\underline{\mathrm{S}}_{\mathrm{t}}{}^{2}$	$\kappa\times{}_{\mathrm{R}}S_{\mathrm{t}}{}^{2}$

Multiplying the relationships above by the metric tensor $g_{\mu\nu}$ of Einstein general relativity, it is

Table 6. Energy, time and space and the four basic fields of nature

		Curvature		
		YES	NO	
Momentum	YES	${}_{0}E_{\mathrm{t}}{}^{2} \times g_{\mu\nu}$	${}_{\mathrm{W}}E_{\mathrm{t}}{}^{2} \times g_{\mu\nu}$	${}_{\mathrm{R}}E_{\mathrm{t}}{}^{2} \times g_{\mu\nu}$
	NO	${}_{0}t_{\mathrm{t}}{}^{2} \times g_{\mu\nu}$	${}_{\mathrm{W}}t_{\mathrm{t}}{}^{2} \times g_{\mu\nu}$	${}_{\mathrm{R}}t_{\mathrm{t}}{}^{2} \times g_{\mu\nu}$
		${}_{0}S_{\mathrm{t}}{}^{2} \times g_{\mu\nu}$	${}_{0}\underline{S}_{\mathrm{t}}{}^{2} \times g_{\mu\nu}$	$\kappa \times {}_{\mathrm{R}}S_{\mathrm{t}}{}^{2} \times g_{\mu\nu}$

Table 6 does not guarantee from the outset neither the dimensional nor other compatibility with Einstein's theory of general relativity.

3.5. The equivalence of time and gravitational field

Theorem 3.13 (The equivalence of time and gravitational field)**.** *(Barukčić, 2011)*

$$E({}_{R}t_{t}) \equiv E({}_{R}g_{t}) \times c^{2} \tag{334}$$

Proof by direct proof. **If** the premise

$$\underbrace{+1=+1}_{(Premise)} \tag{335}$$

is true, **then** the following conclusion

$$E({}_{\mathrm{R}}t_{\mathrm{t}}) \equiv E({}_{\mathrm{R}}g_{\mathrm{t}}) \times c^{2} \tag{336}$$

is also true, again the absence of any technical errors presupposed. The premise

$$+1 \equiv +1 \tag{337}$$

is true. We multiply equation 337 through by equation 303. It is

$$\frac{E({}_{\mathrm{R}}E_{\mathrm{t}})}{{}_{\mathrm{R}}S_{\mathrm{t}}} + \frac{E({}_{\mathrm{R}}t_{\mathrm{t}})}{{}_{\mathrm{R}}S_{\mathrm{t}}} \equiv +1 \tag{338}$$

Equation 338 is rearranged further (see equation 301). It is,

$$\frac{E({}_{\mathrm{R}}E_{\mathrm{t}})}{{}_{\mathrm{R}}S_{\mathrm{t}}} + \frac{E({}_{\mathrm{R}}t_{\mathrm{t}})}{{}_{\mathrm{R}}S_{\mathrm{t}}} \equiv \frac{E({}_{\mathrm{R}}M_{\mathrm{t}})}{{}_{\mathrm{R}}U_{\mathrm{t}}} + \frac{E({}_{\mathrm{R}}g_{\mathrm{t}})}{{}_{\mathrm{R}}U_{\mathrm{t}}} \tag{339}$$

We multiply equation 339 through by ${}_{\mathrm{R}}\mathrm{S}_{\mathrm{t}}$. Equation 339 changes to

$$E({}_{\mathrm{R}}E_{\mathrm{t}}) + E({}_{\mathrm{R}}t_{\mathrm{t}}) \equiv \left(\frac{{}_{\mathrm{R}}S_{\mathrm{t}}}{{}_{\mathrm{R}}U_{\mathrm{t}}} \times E({}_{\mathrm{R}}M_{\mathrm{t}})\right) + \left(\frac{{}_{\mathrm{R}}S_{\mathrm{t}}}{{}_{\mathrm{R}}U_{\mathrm{t}}} \times E({}_{\mathrm{R}}g_{\mathrm{t}})\right) \tag{340}$$

According to equation 302, it is $c^2 \equiv \frac{{}_{\mathrm{R}}S_{\mathrm{t}}}{{}_{\mathrm{R}}U_{\mathrm{t}}}$. Equation 340 changes to

$$E({}_{\mathrm{R}}E_{\mathrm{t}}) + E({}_{\mathrm{R}}t_{\mathrm{t}}) \equiv \left(c^2 \times E({}_{\mathrm{R}}M_{\mathrm{t}})\right) + \left(c^2 \times E({}_{\mathrm{R}}g_{\mathrm{t}})\right) \tag{341}$$

Equation 341 simplifies (see equation 304) as

$$E({}_{\mathrm{R}}E_{\mathrm{t}}) + E({}_{\mathrm{R}}t_{\mathrm{t}}) \equiv E({}_{\mathrm{R}}E_{\mathrm{t}}) + \left(c^2 \times E({}_{\mathrm{R}}g_{\mathrm{t}})\right) \tag{342}$$

Simplifying equation 342, we obtain **the equivalence of time and gravitational field** as

$$E({}_{\mathrm{R}}t_{\mathrm{t}}) \equiv E\left({}_{\mathrm{R}}g_{\mathrm{t}}\right) \times c^2 \tag{343}$$

□

3.6. *Time and gravitational field*

Is there actually any relationship between time and gravitational field and if yes, what kind of relationship could this be?

Theorem 3.14 (Time and gravitational field)**.**

$$E({}_{R}X_{t}) \equiv E({}_{R}t_{t}) \tag{344}$$

Proof by direct proof. **If** the premise

$$\underbrace{+1 = +1}_{(Premise)} \tag{345}$$

is true, **then** the following conclusion

$$E({}_{\mathrm{R}}X_{\mathrm{t}}) \equiv E({}_{\mathrm{R}}t_{\mathrm{t}}) \tag{346}$$

is also true, again the absence of any technical errors presupposed. The premise

$$+1 \equiv +1 \tag{347}$$

is true. We multiply equation 347 through by the gravitational field $\mathrm{E}({}_{\mathrm{R}}\mathrm{g}_{\mathrm{t}})$. It is

$$E({}_{\mathrm{R}}g_{\mathrm{t}}) \equiv E({}_{\mathrm{R}}g_{\mathrm{t}}) \tag{348}$$

Equation 316 has been able to provide clear evidence that there is a third between energy and space, denoted as $E({}_{R}X_{t})$, which is existing independent of any human mind as consciousness, objectively and real. Equation 348 changes to

$$\frac{E({}_{R}X_{t})}{c^{2}} \equiv E\left({}_{R}g_{t}\right) \tag{349}$$

and to

$$E({}_{R}X_{t}) \equiv E\left({}_{R}g_{t}\right) \times c^{2} \tag{350}$$

With the deepest regrets, we had to face the fact that equation 316 has not been able to provide any evidence of the nature of $E({}_{R}X_{t})$ itself. But equation 343 did provide a reliable evidence about the nature of $E({}_{R}X_{t})$. Based on equation 343, it is

$$E({}_{R}X_{t}) \equiv E({}_{R}t_{t}) \tag{351}$$

Time is the unknown third between energy and space which is existing independent of any human mind as consciousness, objectively and real.

□

Theorem 3.15 (The equivalence of time and gravitational field)**.** *The equivalence of time and gravitational field has been proof several times. In this publication, we will choose additionally a new approach to this issue. The equivalence of time and gravitational field is given from the point of view of a stationary observer R by the equation*

$${}_{R}t_{t} \equiv {}_{R}g_{t} \times c^{2} \tag{352}$$

and from the point of view of a co-moving observer 0 by the relationship

$${}_{0}t_{t} \equiv {}_{0}g_{t} \times c^{2} \tag{353}$$

Proof by direct proof. In general, it is (see equations 265, 240 226 and 287)

$$1-\frac{v^{2}}{c^{2}} \equiv \frac{{}_{0}t_{t}{}^{2}}{{}_{R}t_{t}{}^{2}} \equiv \frac{{}_{0}E_{t}{}^{2}}{{}_{R}E_{t}{}^{2}} \equiv \frac{{}_{0}g_{t}{}^{2}}{{}_{R}g_{t}{}^{2}} \tag{354}$$

Dividing equation 354 by c^4 it is

$$\frac{{}_{0}t_{t}{}^{2}}{c^{4}} \times \frac{1}{{}_{R}t_{t}{}^{2}} \equiv \frac{{}_{0}E_{t}{}^{2}}{c^{4}} \times \frac{1}{{}_{R}E_{t}{}^{2}} \equiv \frac{{}_{0}g_{t}{}^{2}}{1} \times \frac{1}{c^{4} \times {}_{R}g_{t}{}^{2}} \tag{355}$$

Multiplying equation 355 by c^4 it is

$$\frac{{}_{0}t_{t}{}^{2}}{c^{4}} \times \frac{c^{4}}{{}_{R}t_{t}{}^{2}} \equiv \frac{{}_{0}E_{t}{}^{2}}{c^{4}} \times \frac{c^{4}}{{}_{R}E_{t}{}^{2}} \equiv \frac{{}_{0}g_{t}{}^{2} \times c^{4}}{1} \times \frac{1}{c^{4} \times {}_{R}g_{t}{}^{2}} \tag{356}$$

Equation 356 is simplified as

$$\frac{{}_{0}t_{t}{}^{2}}{{}_{R}t_{t}{}^{2}} \equiv \frac{{}_{0}m_{t}{}^{2}}{{}_{R}M_{t}{}^{2}} \equiv \frac{{}_{0}g_{t}{}^{2} \times c^{4}}{{}_{R}g_{t}{}^{2} \times c^{4}} \tag{357}$$

Equation before becomes

$$\frac{{}_{0}t_{\mathrm{t}}{}^{2}}{{}_{\mathrm{R}}t_{\mathrm{t}}{}^{2}} \equiv \frac{{}_{0}g_{\mathrm{t}}{}^{2} \times c^{4}}{{}_{\mathrm{R}}g_{\mathrm{t}}{}^{2} \times c^{4}} \tag{358}$$

Taking the square root of equation 358, it is

$$\frac{{}_{0}t_{\mathrm{t}}}{{}_{\mathrm{R}}t_{\mathrm{t}}} \equiv \frac{{}_{0}g_{\mathrm{t}} \times c^{2}}{{}_{\mathrm{R}}g_{\mathrm{t}} \times c^{2}} \tag{359}$$

The equivalence of time and gravitational field is given from the point of view of a co-moving observer 0 by the relationship

$${}_{0}t_{\mathrm{t}} \equiv {}_{0}g_{\mathrm{t}} \times c^{2} \tag{360}$$

and from the point of view of a stationary observer R by the equation

$${}_{\mathrm{R}}t_{\mathrm{t}} \equiv {}_{\mathrm{R}}g_{\mathrm{t}} \times c^{2} \tag{361}$$

Equation 229 becomes (see equation 361)

$$({}_{\mathrm{R}}S_{\mathrm{t}}) \equiv +({}_{\mathrm{R}}E_{\mathrm{t}}) + \left({}_{\mathrm{R}}g_{\mathrm{t}} \times c^{2}\right) \equiv +({}_{\mathrm{R}}E_{\mathrm{t}}) + ({}_{\mathrm{R}}t_{\mathrm{t}}) \tag{362}$$

□

3.7. The scalar of space S

It is appropriate to consider that invariants (scalars) are tensors of rank 0 (Einstein, 1923b, p. 13).

Theorem 3.16 (The scalar ${}_{\mathrm{R}}\mathrm{S}_{\mathrm{t}}$)**.** *In general, the Scalar ${}_{R}S_{t}$ is determined as*

$$\kappa \times {}_{R}S_{t}{}^{2} \equiv \frac{R}{D} \tag{363}$$

while R is the Ricci curvature scalar (see definition 2.44), D is defined (see definition 50) as $g_{\mu\nu} \times g^{\mu\nu} \equiv D$ and $\kappa \times {}_{R}S_{t}{}^{2}$ might denote even something like (the 'density' of) space.

Proof by direct proof. **If** the premise

$$\underbrace{+1 = +1}_{(Premise)} \tag{364}$$

is true, **then** the conclusion

$$R \equiv \left(\kappa \times {}_{\mathrm{R}}S_{\mathrm{t}}{}^{2}\right) \times D \tag{365}$$

is also true, the absence of any technical errors presupposed. The premise or respectively axiom I

$$+1 \equiv +1 \tag{366}$$

is true. Multiplying this premise (i.e. axiom I) by the Ricci tensor $R_{\mu\nu}$, it is

$$R_{\mu\nu} \equiv R_{\mu\nu} \tag{367}$$

The general form of the Ricci tensor is determined (see equation 152) as $R_{\mu\nu} \equiv \left(\kappa \times {}_{\mathrm{R}}S_{\mathrm{t}}{}^{2}\right) \times g_{\mu\nu}$ which is substituted into equation 367.

$$R_{\mu\nu} \equiv \left(\kappa \times {}_{\mathrm{R}}S_{\mathrm{t}}{}^{2}\right) \times g_{\mu\nu} \tag{368}$$

We multiply equation 368 through by the inverse metric tensor $g^{\mu\nu}$. It is

$$R_{\mu\nu} \times g^{\mu\nu} \equiv \left(\kappa \times {}_{\mathrm{R}}S_{\mathrm{t}}{}^{2}\right) \times g_{\mu\nu} \times g^{\mu\nu} \tag{369}$$

In general, the Ricci scalar R is determined as

$$R \equiv \left(\kappa \times {}_{\mathrm{R}}S_{\mathrm{t}}{}^{2}\right) \times D \equiv (\kappa \times D) \times {}_{\mathrm{R}}S_{\mathrm{t}}{}^{2} \tag{370}$$

In general, the scalar S is determined without an exception as

$$\kappa \times {}_{\mathrm{R}}S_{\mathrm{t}}{}^{2} \equiv \frac{R}{D} \tag{371}$$

□

3.8. *The parameter x_3*

It can be assumed that special theory of relativity relies more or less on the gravitational field, while Einstein's general theory of relativity is working with the gravitational potential.

Theorem 3.17 (The scalar of sapce and the Ricci scalar)**.** *Under conditions where an additional correction factor x_3 is needed to assure complete compatibility with Einstein's general theory of relativity, it is*

$$x_3 \times \kappa \times {}_{R}S_{t}{}^{2} \equiv \frac{R}{D} \tag{372}$$

while R is the Ricci curvature scalar (see definition 2.44), D is defined (see definition 50) as $g_{\mu\nu} \times g^{\mu\nu} \equiv D$ and $x_3 \times \kappa \times {}_{R}S_{t}{}^{2}$ might denote even something like (the 'density'of) space.

Proof by direct proof. **If** the premise

$$\underbrace{+1 = +1}_{(Premise)} \tag{373}$$

is true, **then** the conclusion

$$R \equiv \left(x_3 \times \kappa \times {}_{\mathrm{R}}S_{\mathrm{t}}{}^{2}\right) \times D \tag{374}$$

is also true, the absence of any technical errors presupposed. The premise or respectively axiom I

$$+1 \equiv +1 \tag{375}$$

is true. Multiplying this premise (i.e. axiom I) by the Ricci tensor $R_{\mu\nu}$, it is

$$R_{\mu\nu} \equiv R_{\mu\nu} \tag{376}$$

The general form of the Ricci tensor is determined (see equation 152) as $R_{\mu\nu} \equiv \left(x_3 \times \kappa \times {_{\mathrm{R}}S_{\mathrm{t}}}^2\right) \times g_{\mu\nu}$ which is substituted into equation 367.

$$R_{\mu\nu} \equiv \left(x_3 \times \kappa \times {_{\mathrm{R}}S_{\mathrm{t}}}^2\right) \times g_{\mu\nu} \tag{377}$$

We multiply equation 368 through by the inverse metric tensor $g^{\mu\nu}$. It is

$$R_{\mu\nu} \times g^{\mu\nu} \equiv \left(x_3 \times \kappa \times {_{\mathrm{R}}S_{\mathrm{t}}}^2\right) \times g_{\mu\nu} \times g^{\mu\nu} \tag{378}$$

In general, the Ricci scalar R is determined as

$$R \equiv \left(x_3 \times \kappa \times {_{\mathrm{R}}S_{\mathrm{t}}}^2\right) \times D \equiv (x_3 \times \kappa \times D) \times {_{\mathrm{R}}S_{\mathrm{t}}}^2 \tag{379}$$

The scalar S is determined as

$$x_3 \times \kappa \times {_{\mathrm{R}}S_{\mathrm{t}}}^2 \equiv \frac{R}{D} \tag{380}$$

There are circumstances where $x_3 = 1$. The following table 7 might illustrate this situation.

Table 7. Energy, time and space and the four basic fields of nature

		Curvature		
		YES	NO	
Momentum	YES	$\left(x_3 \times {_0E_{\mathrm{t}}}^2 \times g_{\mu\nu}\right)$	$\left(x_3 \times {_{\mathrm{W}}E_{\mathrm{t}}}^2 \times g_{\mu\nu}\right)$	$\left(x_3 \times {_{\mathrm{R}}E_{\mathrm{t}}}^2 \times g_{\mu\nu}\right)$
	NO	$\left(x_3 \times {_0t_{\mathrm{t}}}^2 \times g_{\mu\nu}\right)$	$\left(x_3 \times {_{\mathrm{W}}t_{\mathrm{t}}}^2 \times g_{\mu\nu}\right)$	$\left(x_3 \times {_{\mathrm{R}}t_{\mathrm{t}}}^2 \times g_{\mu\nu}\right)$
		$\left(x_3 \times {_0S_{\mathrm{t}}}^2 \times g_{\mu\nu}\right)$	$\left(x_3 \times {_0\underline{S}_{\mathrm{t}}}^2 \times g_{\mu\nu}\right)$	$\left(x_3 \times \kappa \times {_{\mathrm{R}}S_{\mathrm{t}}}^2 \times g_{\mu\nu}\right)$

It is feasible that in exceptional circumstances the following relationships might hold true:

$$x_3 \times \kappa \times {_{\mathrm{R}}S_{\mathrm{t}}}^2 \equiv \frac{R}{D} \tag{381}$$

and at the end that

$$x_3 \equiv \frac{R}{D \times \kappa \times {_{\mathrm{R}}S_{\mathrm{t}}}^2} \tag{382}$$

□

However, even the relationship

$$x_3 \times \kappa \times {_{\mathrm{R}}S_{\mathrm{t}}}^2 \equiv \frac{R}{D} \equiv \Psi \tag{383}$$

is possible too. The complex conjugate Ψ^*, in order to achieve normalization like $\Psi \times \Psi^* \equiv +1$, can follow as,

$$\Psi^* \equiv \frac{1}{x_3 \times \kappa \times {}_{\mathrm{R}}S_{\mathrm{t}}{}^2} \equiv \frac{D}{R} \tag{384}$$

However, these thoughts are purely theoretical and of provisional in nature.

3.9. The parameter x_5 I

Theorem 3.18 (The parameter x_5 I). *In general, it is*

$$ {}_{R}E_{t}{}^{2} \equiv x_5 \times \left(\left(\frac{R}{D} \right) - \left(\frac{R}{2} \right) + \Lambda \right) \equiv x_5 \times \left(\frac{8 \times \pi \times \gamma \times T}{c^4 \times D} \right) \equiv x_5 \times {}_{d}E_{t}{}^{2} \tag{385}$$

Proof by direct proof. The premise

$$+1 \equiv +1 \tag{386}$$

is true. In the following, we rearrange the premise before. We obtain

$${}_{\mathrm{R}}E_{\mathrm{t}}{}^2 \equiv {}_{\mathrm{R}}E_{\mathrm{t}}{}^2 \tag{387}$$

Equation 387 changes (see equation 154). It is

$${}_{\mathrm{R}}E_{\mathrm{t}}{}^2 \equiv {}_{\mathrm{R}}E_{\mathrm{t}}{}^2 \times 1 \equiv {}_{\mathrm{R}}E_{\mathrm{t}}{}^2 \times \frac{\left(\frac{R}{D} \right) - \left(\frac{R}{2} \right) + \Lambda}{\left(\frac{R}{D} \right) - \left(\frac{R}{2} \right) + \Lambda} \tag{388}$$

and equally

$${}_{\mathrm{R}}E_{\mathrm{t}}{}^2 \equiv \frac{{}_{\mathrm{R}}E_{\mathrm{t}}{}^2}{\left(\left(\frac{R}{D} \right) - \left(\frac{R}{2} \right) + \Lambda \right)} \times \left(\left(\frac{R}{D} \right) - \left(\frac{R}{2} \right) + \Lambda \right) \tag{389}$$

In our understanding, it is $x_5 \equiv \frac{{}_{\mathrm{R}}E_{\mathrm{t}}{}^2}{\left(\left(\frac{R}{D} \right) - \left(\frac{R}{2} \right) + \Lambda \right)}$. Equation 389 becomes

$${}_{\mathrm{R}}E_{\mathrm{t}}{}^2 \equiv x_5 \times \left(\left(\frac{R}{D} \right) - \left(\frac{R}{2} \right) + \Lambda \right) \equiv x_5 \times \left(\frac{8 \times \pi \times \gamma \times T}{c^4 \times D} \right) \equiv x_5 \times {}_{\mathrm{d}}E_{\mathrm{t}}{}^2 \tag{390}$$

□

However, another straightforward consequence of equation 390 is the relationship

$$\left(\frac{8 \times \pi \times \gamma \times T}{c^4 \times D} \right) \equiv \left(\left(\frac{R}{D} \right) - \left(\frac{R}{2} \right) + \Lambda \right) \equiv \frac{{}_{\mathrm{R}}E_{\mathrm{t}}{}^2}{x_5} \equiv {}_{\mathrm{d}}E_{\mathrm{t}}{}^2 \tag{391}$$

3.10. The parameter x_5 II

Theorem 3.19 (The term x_5)**.** *There are circumstances under which x_5 is determined as*

$$x_5 \equiv \frac{D \times \kappa \times {}_R S_t{}^2}{R} \tag{392}$$

Proof by direct proof. The premise

$$+1 \equiv +1 \tag{393}$$

is true. Multiplying by $G_{\mu\nu}$, it is

$$G_{\mu\nu} \equiv G_{\mu\nu} \tag{394}$$

or

$$G_{\mu\nu} + (\Lambda \times g_{\mu\nu}) \equiv G_{\mu\nu} + (\Lambda \times g_{\mu\nu}) \tag{395}$$

and equally

$$R_{\mu\nu} - \left(\frac{R}{2} \times g_{\mu\nu}\right) + (\Lambda \times g_{\mu\nu}) \equiv G_{\mu\nu} + (\Lambda \times g_{\mu\nu}) \tag{396}$$

Einstein's field equations are defined as $G_{\mu\nu} + (\Lambda \times g_{\mu\nu}) = \left(\frac{8 \times \pi \times \gamma}{c^4}\right) \times T_{\mu\nu}$ Equation 396 becomes

$$R_{\mu\nu} - \left(\frac{R}{2} \times g_{\mu\nu}\right) + (\Lambda \times g_{\mu\nu}) \equiv \left(\frac{8 \times \pi \times \gamma}{c^4}\right) \times T_{\mu\nu} \tag{397}$$

Equation 397 changes to

$$\left(\left(\frac{8 \times \pi \times \gamma}{c^4}\right) \times T_{\mu\nu}\right) + \left(\frac{R}{2} \times g_{\mu\nu}\right) - (\Lambda \times g_{\mu\nu}) \equiv R_{\mu\nu} \tag{398}$$

In our understanding it is

$$\left({}_f a^2 \times g_{\mu\nu}\right) + \left({}_f b^2 \times g_{\mu\nu}\right) + \left({}_f c^2 \times g_{\mu\nu}\right) + \left({}_f d^2 \times g_{\mu\nu}\right) \equiv R_{\mu\nu} \tag{399}$$

In other words, equation 398 is equivalent (see equation 581) with the relationship

$$\begin{aligned} &\left(\left(\frac{8 \times \pi \times \gamma \times T}{c^4 \times D}\right) \times g_{\mu\nu}\right) + \left(\frac{R}{2} \times g_{\mu\nu}\right) - (\Lambda \times g_{\mu\nu}) \\ &\equiv \frac{R}{D} \times g_{\mu\nu} \\ &\equiv \left({}_f a^2 \times g_{\mu\nu}\right) + \left({}_f b^2 \times g_{\mu\nu}\right) + \left({}_f c^2 \times g_{\mu\nu}\right) + \left({}_f d^2 \times g_{\mu\nu}\right) \end{aligned} \tag{400}$$

Multiplying the Einstein field equation (see equation 400) by the term x_5, it is

$$\begin{aligned} &\left(x_5 \times \left(\frac{8 \times \pi \times \gamma \times T}{c^4 \times D}\right) \times g_{\mu\nu}\right) + \left(x_5 \times \frac{R}{2} \times g_{\mu\nu}\right) - (x_5 \times \Lambda \times g_{\mu\nu}) \\ &\equiv x_5 \times \frac{R}{D} \times g_{\mu\nu} \\ &\equiv \left(x_5 \times {}_f a^2 \times g_{\mu\nu}\right) + \left(x_5 \times {}_f b^2 \times g_{\mu\nu}\right) + \left(x_5 \times {}_f c^2 \times g_{\mu\nu}\right) + \left(x_5 \times {}_f d^2 \times g_{\mu\nu}\right) \end{aligned} \tag{401}$$

These relationships are illustrated by table 8.

Table 8. Energy, time and space and the four basic fields of nature

		Curvature YES		NO
Momentum	YES	$\left(x_5 \times {}_{\mathrm{f}}a^2 \times g_{\mu\nu}\right)$	$\left(x_5 \times {}_{\mathrm{f}}b^2 \times g_{\mu\nu}\right)$	$\left(x_5 \times {}_{\mathrm{d}}E_{\mathrm{t}}{}^2 \times g_{\mu\nu}\right)$
	NO	$\left(x_5 \times {}_{\mathrm{f}}c^2 \times g_{\mu\nu}\right)$	$\left(x_5 \times {}_{\mathrm{f}}d^2 \times g_{\mu\nu}\right)$	$\left(x_5 \times {}_{\mathrm{d}}\underline{E}^2 \times g_{\mu\nu}\right)$
		$\left(x_5 \times {}_{\mathrm{d}}G^2 \times g_{\mu\nu}\right)$	$\left(x_5 \times {}_{\mathrm{d}}\underline{G}^2 \times g_{\mu\nu}\right)$	$\left(x_5 \times \frac{R}{D} \times g_{\mu\nu}\right)$

In our understanding, it is

$$ {}_{0}E_{\mathrm{t}}{}^2 \times g_{\mu\nu} \equiv \left(x_5 \times {}_{\mathrm{f}}a^2 \times g_{\mu\nu}\right) \tag{402} $$

and

$$ {}_{0}t_{\mathrm{t}}{}^2 \times g_{\mu\nu} \equiv \left(x_5 \times {}_{\mathrm{f}}c^2 \times g_{\mu\nu}\right) \tag{403} $$

and

$$ {}_{\mathrm{W}}E_{\mathrm{t}}{}^2 \times g_{\mu\nu} \equiv \left(x_5 \times {}_{\mathrm{f}}b^2 \times g_{\mu\nu}\right) \tag{404} $$

and

$$ {}_{\mathrm{W}}t_{\mathrm{t}}{}^2 \times g_{\mu\nu} \equiv \left(x_5 \times {}_{\mathrm{f}}d^2 \times g_{\mu\nu}\right) \tag{405} $$

Furthermore, it is

$$ {}_{\mathrm{R}}t_{\mathrm{t}}{}^2 \times g_{\mu\nu} \equiv \left(x_5 \times \left(\frac{R}{2} - \Lambda\right) \times g_{\mu\nu}\right) \tag{406} $$

and

$$ {}_{\mathrm{R}}E_{\mathrm{t}}{}^2 \times g_{\mu\nu} \equiv \left(x_5 \times \left(\frac{8 \times \pi \times \gamma \times T}{c^4 \times D}\right) \times g_{\mu\nu}\right) \equiv x_5 \times \left(\frac{R}{D} - \left(\frac{R}{2}\right) + (\Lambda)\right) \times g_{\mu\nu} \equiv x_5 \times {}_{\mathrm{d}}E_{\mathrm{t}}{}^2 \times g_{\mu\nu} \tag{407} $$

Taking the trace of equation 401, it is

$$ \left(x_5 \times \left(\frac{8 \times \pi \times \gamma \times T}{c^4 \times D}\right) \times g_{\mu\nu} \times g^{\mu\nu}\right) + \left(x_5 \times \frac{R}{2} \times g_{\mu\nu} \times g^{\mu\nu}\right) - \left(x_5 \times \Lambda \times g_{\mu\nu} \times g^{\mu\nu}\right) \equiv x_5 \times \frac{R}{D} \times g_{\mu\nu} \times g^{\mu\nu} \tag{408} $$

or (see equation 50)

$$ \left(x_5 \times \left(\frac{8 \times \pi \times \gamma \times T}{c^4 \times D}\right) \times D\right) + \left(x_5 \times \frac{R}{2} \times D\right) - \left(x_5 \times \Lambda \times D\right) \equiv x_5 \times \frac{R}{D} \times D \tag{409} $$

Equation 409 simplifies as

$$\left(x_5 \times \left(\frac{8 \times \pi \times \gamma \times T}{c^4}\right)\right) + \left(x_5 \times \frac{R}{2} \times D\right) - (x_5 \times \Lambda \times D) \equiv x_5 \times \frac{R}{1} \tag{410}$$

Dividing equation 410 by the term D, it is

$$\left(x_5 \times \underbrace{\left(\frac{8 \times \pi \times \gamma \times T}{c^4 \times D}\right)}_{{}_{\mathrm{d}}E_{\mathrm{t}}{}^2}\right) + \left(x_5 \times \frac{R}{2}\right) - (x_5 \times \Lambda) \equiv x_5 \times \frac{R}{D} \tag{411}$$

From equation 411 follows that

$${}_{\mathrm{R}}E_{\mathrm{t}}{}^2 \equiv x_5 \times {}_{\mathrm{d}}E_{\mathrm{t}}{}^2 \equiv x_5 \times \left(\frac{R}{D} - \left(\frac{R}{2}\right) + (\Lambda)\right) \tag{412}$$

Under conditions where

$$x_5 \times \frac{R}{D} \equiv \kappa \times {}_{\mathrm{R}}S_{\mathrm{t}}{}^2 \tag{413}$$

it is

$$x_5 \equiv \frac{D \times \kappa \times {}_{\mathrm{R}}S_{\mathrm{t}}{}^2}{R} \tag{414}$$

□

Equation 414 does not exclude circumstances (see equation 371) where $x_5 = +1$. Under these conditions, it is

$$R \equiv D \times \kappa \times {}_{\mathrm{R}}S_{\mathrm{t}}{}^2 \tag{415}$$

It is worthwhile noting that

$$x_3 \times x_5 \equiv +1 \tag{416}$$

3.11. The geometrical form of the stress-energy tensor of the electromagnetic field

Theorem 3.20 (The geometrical form of the stress-energy tensor of the electromagnetic field $b_{\mu\nu}$). *The geometrization of the stress-energy tensor of the electromagnetic fields has been left behind by Einstein (Einstein, 1916) himself as an unsolved problem. Besides of the many trials to extend the geometry of general relativity even to the electromagnetic field, the conceptual differences between the geometrized gravitational field and the classical Maxwellian theory of the electromagnetic filed were so far insurmountable.*

CLAIM.

In general, the completely geometrical form of the stress-energy momentum tensor of the electromagnetic field $b_{\mu\nu}$ depending upon metric signature is given by

$$b_{\mu\nu} \equiv \left(\frac{(4 \times F_c) + (D \times F_e)}{4 \times \pi \times 4 \times D}\right) \times g_{\mu\nu} \tag{417}$$

Proof by modus ponens. **If** the premise

$$\underbrace{+1 = +1}_{(Premise)} \tag{418}$$

is true, **then** the conclusion

$$\begin{aligned} b_{\mu\nu} &\equiv \left(\frac{(4 \times F_{\mathrm{c}}) + (D \times F_{\mathrm{e}})}{4 \times \pi \times 4 \times D}\right) \times g_{\mu\nu} \\ &\equiv b \times g_{\mu\nu} \end{aligned} \tag{419}$$

is also true, the absence of any technical errors presupposed. The premise

$$(+1) = (+1) \tag{420}$$

is true. Multiplying this premise by the stress-energy momentum tensor of the electromagnetic field $b_{\mu\nu}$, we obtain

$$(+1) \times b_{\mu\nu} \equiv (+1) \times b_{\mu\nu} \tag{421}$$

or

$$b_{\mu\nu} \equiv b_{\mu\nu} \tag{422}$$

The tensor $b_{\mu\nu}$ denotes the trace-less, symmetric stress-energy tensor of the (source-free) electromagnetic field and is defined (see Lehmkuhl, 2011, p. 13) (**depending upon metric signature (see Hughston and Tod, 1990, p. 38)**) as

$$b_{\mu\nu} \equiv \left(\frac{1}{4 \times \pi} \times \left(\left(F_{\mu\,\mathrm{c}} \times F_{\nu}{}^{\mathrm{c}}\right) + \left(\frac{1}{4} \times g_{\mu\nu} \times F_{\mathrm{de}} \times F^{\mathrm{de}}\right)\right)\right) \tag{423}$$

A completely geometrized, co-variant stress-energy tensor of the electromagnetic field expressed under conditions of D = 4 space-time dimensions has already been published (theorem 3.1, equation 80 Barukčić, 2020a, p. 157). Rearranging equation 423 in connection with equation 142 and according to the definition 2.65 it is

$$b_{\mu\nu} \equiv \left(\frac{1}{4 \times \pi} \times \left(\left(F_{\mu\,\mathrm{c}} \times F_{\nu\,\mathrm{d}} \times g^{\mathrm{cd}}\right) + \left(\frac{1}{4} \times g_{\mu\nu} \times F_{\mathrm{de}} \times F^{\mathrm{de}}\right)\right)\right) \tag{424}$$

Rearranging equation 424, it is

$$b_{\mu\nu} \equiv \frac{1}{4 \times \pi} \times \left(\left(\frac{4 \times D}{4 \times D} \times \left(F_{\mu\,\mathrm{c}} \times F_{\nu\,\mathrm{d}} \times g^{\mathrm{cd}}\right)\right) + \left(\left(\frac{D}{4 \times D} \times F_{\mathrm{de}} \times F^{\mathrm{de}}\right) \times g_{\mu\nu}\right)\right) \tag{425}$$

where D denotes the number of space-time dimensions (see definition 2.12, equation 50). Rearranging the equation 425 further, we obtain

$$b_{\mu\nu} \equiv \frac{1}{4\times\pi\times4\times D}\times\left(\left(4\times D\times\left(F_{\mu\,\mathrm{c}}\times F_{\nu\,\mathrm{d}}\times g^{\mathrm{cd}}\right)\right)+\left(D\times\left(F_{\mathrm{de}}\times F^{\mathrm{de}}\right)\times g_{\mu\nu}\right)\right) \tag{426}$$

Under conditions where $g_{\mu\nu}\times g^{\mu\nu}\equiv D$ (see definition 2.12, equation 50) equation 426 simplifies as

$$b_{\mu\nu} \equiv \frac{1}{4\times\pi\times4\times D}\times\left(\left(4\times\left(g_{\mu\nu}\times g^{\mu\nu}\right)\times\left(F_{\mu\,\mathrm{c}}\times F_{\nu\,\mathrm{d}}\times g^{\mathrm{cd}}\right)\right)+\left(D\times\left(F_{\mathrm{de}}\times F^{\mathrm{de}}\right)\times g_{\mu\nu}\right)\right) \tag{427}$$

or as

$$b_{\mu\nu} \equiv \frac{1}{4\times\pi\times4\times D}\times\left(\left(\left(4\times\left(g^{\mu\nu}\right)\times\left(F_{\mu\,\mathrm{c}}\times F_{\nu\,\mathrm{d}}\times g^{\mathrm{cd}}\right)\right)\times g_{\mu\nu}\right)+\left(D\times\left(F_{\mathrm{de}}\times F^{\mathrm{de}}\right)\times g_{\mu\nu}\right)\right) \tag{428}$$

A further simplification of the relationship before (equation 428) yields the stress-energy momentum tensor of the electromagnetic field $\mathrm{b}_{\mu\nu}$ determined only by the metric tensor of general relativity $\mathrm{g}_{\mu\nu}$ as

$$b_{\mu\nu} \equiv \frac{1}{4\times\pi\times4\times D}\times\left(\left(4\times\left(F_{\mu\,\mathrm{c}}\times g^{\mu\nu}\times g^{\mathrm{cd}}\times F_{\nu\,\mathrm{d}}\right)\right)+\left(D\times\left(F_{\mathrm{de}}\times F^{\mathrm{de}}\right)\right)\right)\times g_{\mu\nu} \tag{429}$$

However, the term $\left(\left(F_{\mu\,\mathrm{c}}\times g^{\mu\nu}\times g^{\mathrm{cd}}\times F_{\nu\,\mathrm{d}}\right)+\left(F_{\mathrm{de}}\times F^{\mathrm{de}}\right)\right)$ of the equation 429 can be simplified further. For the first, we define (see definition 2.63, equation 170) the invariant

$$F_{\mathrm{e}} \equiv F_{\mathrm{de}}\times F^{\mathrm{de}} \tag{430}$$

Furthermore, for an order-2 tensor, twice multiplying by the contra-variant metric tensor and contracting (see Einstein, 1916, p. 790) in different indices (see Kay, 1988) raises each index. In other (Jackson, 1975) words, according to Einstein (see Einstein, 1916, p. 790), it is in general $F^{\left(\begin{smallmatrix}1&3\\ \mu&c\end{smallmatrix}\right)}\equiv g^{\left(\begin{smallmatrix}1&2\\ \mu&\nu\end{smallmatrix}\right)}\times g^{\left(\begin{smallmatrix}3&4\\ c&d\end{smallmatrix}\right)}\times F_{\left(\begin{smallmatrix}\nu&d\\ 2&4\end{smallmatrix}\right)}$ or more professionally $F^{\mu\,\mathrm{c}}\equiv g^{\mu\nu}\times g^{\mathrm{cd}}\times F_{\nu\,\mathrm{d}}$ (see definition 2.17, equation 62) which simplifies equation 429 as

$$b_{\mu\nu} \equiv \frac{1}{4\times\pi\times4\times D}\times\left(\left(4\times\left(F_{\mu\,\mathrm{c}}\times F^{\mu\,\mathrm{c}}\right)\right)-\left(D\times\left(F_{\mathrm{e}}\right)\right)\right)\times g_{\mu\nu} \tag{431}$$

We define (see definition 2.63, equation 170) the invariant

$$F_{\mathrm{c}} \equiv F_{\mu\,\mathrm{c}}\times F^{\mu\,\mathrm{c}} \tag{432}$$

Equation 431 can be simplified further. It is

$$b_{\mu\nu} \equiv \frac{1}{4\times\pi\times4\times D}\times\left(\left(4\times\left(F_{\mathrm{c}}\right)\right)-\left(D\times\left(F_{\mathrm{e}}\right)\right)\right)\times g_{\mu\nu} \tag{433}$$

or

$$b_{\mu\nu} \equiv \left(\frac{1}{4\times\pi}\times\left(\left(\frac{4\times F_{\mathrm{c}}}{4\times D}\right)+\left(\frac{D\times F_{\mathrm{e}}}{4\times D}\right)\right)\right)\times g_{\mu\nu} \tag{434}$$

and finally as

$$b_{\mu\nu} \equiv \left(\frac{(4 \times F_{\text{c}}) + (D \times F_{\text{e}})}{4 \times \pi \times 4 \times D}\right) \times g_{\mu\nu} \tag{435}$$

Under conditions of D space-time dimensions where

$$\left(F_{\mu\,\text{c}} \times F^{\mu\,\text{c}}\right) \equiv \left(F_{\text{de}} \times F^{\text{de}}\right) \equiv F_{\text{c}} \equiv F_{\text{e}} \tag{436}$$

equation 434 simplifies as

$$b_{\mu\nu} \equiv \left(\frac{1}{4 \times \pi} \times \left(\left(\frac{4 \times F_{\text{e}}}{4 \times D}\right) + \left(\frac{D \times F_{\text{e}}}{4 \times D}\right)\right)\right) \times g_{\mu\nu} \tag{437}$$

or as

$$b_{\mu\nu} \equiv \left(\frac{(4 + D) \times F_{\text{e}}}{4 \times \pi \times 4 \times D}\right) \times g_{\mu\nu} \tag{438}$$

In general, we define the scalar or invariant b as

$$b \equiv \left(\frac{(4 \times F_{\text{c}}) + (D \times F_{\text{e}})}{4 \times \pi \times 4 \times D}\right) \tag{439}$$

The 2-index stress-energy momentum tensor of the electromagnetic field geometrized completely, is given by

$$b_{\mu\nu} \equiv b \times g_{\mu\nu} \tag{440}$$

In other words, our conclusion is true.

□

Remark 3.1. *Under the circumstances before (see definition 2.63, equation 170), the unification of the strong force and weak force is supported and demanded by the stress energy tensor of ordinary matter $a_{\mu\nu}$ as*

$$\begin{aligned} a_{\mu\nu} &\equiv E_{\mu\nu} - b_{\mu\nu} \\ &\equiv \left(\left(\frac{8 \times \pi \times \gamma \times T}{c^4 \times D}\right) - \left(\frac{(4 \times F_c) + (D \times F_e)}{4 \times \pi \times 4 \times D}\right)\right) \times g_{\mu\nu} \end{aligned} \tag{441}$$

Theorem 3.21 (The geometrical form of the stress energy tensor of the electromagnetic field ${}_{\text{W}}\text{E}_{\mu\nu}$). *The geometrical form of the stress energy tensor of the electromagnetic field, denoted by*

$$\begin{aligned} b_{\mu\nu} &\equiv b \times g_{\mu\nu} \\ &\equiv {}_{f}b^2 \times g_{\mu\nu} \\ &\equiv \left(\frac{v^2}{c^2}\right) \times \left(\frac{R}{D} - \left(\frac{R}{2}\right) + (\Lambda)\right) \times g_{\mu\nu} \\ &\equiv \left(\frac{v^2}{c^2}\right) \times \left(\frac{8 \times \pi \times \gamma \times T}{c^4 \times D}\right) \times g_{\mu\nu} \\ &\equiv {}_{W}E_{\mu\nu} \end{aligned} \tag{442}$$

Proof by direct proof. The premise

$$+1 \equiv +1 \tag{443}$$

is true. Multiplying equation 443 by the energy of the electromagnetic wave, ${}_{\mathrm{W}}E_{\mathrm{t}}{}^{2}$, it is

$${}_{\mathrm{W}}E_{\mathrm{t}}{}^{2} \equiv {}_{\mathrm{W}}E_{\mathrm{t}}{}^{2} \tag{444}$$

Dividing equation 444 by the total energy of a system, ${}_{\mathrm{R}}E_{\mathrm{t}}{}^{2}$, it is

$$\frac{{}_{\mathrm{W}}E_{\mathrm{t}}{}^{2}}{{}_{\mathrm{R}}E_{\mathrm{t}}{}^{2}} \equiv \frac{{}_{\mathrm{W}}E_{\mathrm{t}}{}^{2}}{{}_{\mathrm{R}}E_{\mathrm{t}}{}^{2}} \tag{445}$$

In accordance with equation 254, equation 445 becomes

$$\frac{{}_{\mathrm{W}}E_{\mathrm{t}}{}^{2}}{{}_{\mathrm{R}}E_{\mathrm{t}}{}^{2}} \equiv \frac{v^{2}}{c^{2}} \tag{446}$$

Equation 446 changes to (see equation 412)

$$\frac{{}_{\mathrm{W}}E_{\mathrm{t}}{}^{2}}{x_{5} \times \left(\frac{R}{D} - \left(\frac{R}{2}\right) + (\Lambda)\right)} \equiv \frac{v^{2}}{c^{2}} \tag{447}$$

and to

$$\frac{{}_{\mathrm{W}}E_{\mathrm{t}}{}^{2}}{x_{5}} \equiv \left(\frac{v^{2}}{c^{2}}\right) \times \left(\frac{R}{D} - \left(\frac{R}{2}\right) + (\Lambda)\right) \tag{448}$$

Multiplying equation 448 by the metric tensor $g_{\mu\nu}$ of general theory of relativity, it is

$$\frac{{}_{\mathrm{W}}E_{\mathrm{t}}{}^{2}}{x_{5}} \times g_{\mu\nu} \equiv \left(\frac{v^{2}}{c^{2}}\right) \times \left(\frac{R}{D} - \left(\frac{R}{2}\right) + (\Lambda)\right) \times g_{\mu\nu} \tag{449}$$

As illustrated by table 8, it is

$${}_{\mathrm{W}}E_{\mathrm{t}}{}^{2} \times g_{\mu\nu} \equiv \left(x_{5} \times b \times g_{\mu\nu}\right) \tag{450}$$

and

$$\frac{{}_{\mathrm{W}}E_{\mathrm{t}}{}^{2}}{x_{5}} \times g_{\mu\nu} \equiv \left(b \times g_{\mu\nu}\right) \tag{451}$$

Substituting the result of the equation 451 into equation 449, the general form of the stress energy tensor of the electromagnetic field is given by

$$\begin{aligned}
b_{\mu\nu} &\equiv b \times g_{\mu\nu} \\
&\equiv {}_{\mathrm{f}}b^{2} \times g_{\mu\nu} \\
&\equiv \left(\frac{v^{2}}{c^{2}}\right) \times \left(\frac{R}{D} - \left(\frac{R}{2}\right) + (\Lambda)\right) \times g_{\mu\nu} \\
&\equiv \left(\frac{v^{2}}{c^{2}}\right) \times \left(\frac{8 \times \pi \times \gamma \times T}{c^{4} \times D}\right) \times g_{\mu\nu} \\
&\equiv {}_{\mathrm{W}}E_{\mu\nu}
\end{aligned} \tag{452}$$

□

Theorem 3.22 (The geometrical form of the stress energy tensor of the electromagnetic field $_{\mathrm{W}}\mathrm{E}_{\mu\nu}$ I)**.**
In general, it is

$$b \times g_{\mu\nu} \equiv \left(\frac{1}{4 \times \pi \times D} \times \left(\left(F_{\mu\,c} \times F^{\mu\,c}\right) + \left(\frac{D}{4} \times F_{de} \times F^{de}\right)\right)\right) \times g_{\mu\nu} \tag{453}$$

Proof by direct proof. At the beginning of this theorem, it is necessary and appropriate that an important point is being made about the theoretical starting point. All the subsequent content of this theorem stems from premise (i.e. axiom)

$$+1 \equiv +1 \tag{454}$$

Rearranging the equation before, it is

$$b \times g_{\mu\nu} \equiv b \times g_{\mu\nu} \tag{455}$$

Equation 455 changes (see equation 142, p. 38) slightly to

$$b \times g_{\mu\nu} \equiv \left(\frac{1}{4 \times \pi} \times \left(\left(F_{\mu\,\mathrm{c}} \times F_{\nu}{}^{\mathrm{c}}\right) + \left(\frac{1}{4} \times g_{\mu\nu} \times F_{\mathrm{de}} \times F^{\mathrm{de}}\right)\right)\right) \tag{456}$$

It is $g_{\mu\nu} \times g^{\mu\nu} \equiv D$ (see equation 50, p. 16), where D might denote the number of space-time dimensions. Equation 456 becomes

$$b \times g_{\mu\nu} \equiv \left(\frac{1}{4 \times \pi} \times \left(\left(F_{\mu\,\mathrm{c}} \times F_{\nu}{}^{\mathrm{c}}\right) + \left(\frac{1}{4} \times g_{\mu\nu} \times F_{\mathrm{de}} \times F^{\mathrm{de}}\right)\right)\right) \times \frac{D}{D} \tag{457}$$

or

$$b \times g_{\mu\nu} \equiv \left(\frac{1}{4 \times \pi \times D} \times \left(\left(F_{\mu\,\mathrm{c}} \times F_{\nu}{}^{\mathrm{c}}\right) + \left(\frac{1}{4} \times g_{\mu\nu} \times F_{\mathrm{de}} \times F^{\mathrm{de}}\right)\right)\right) \times D \tag{458}$$

In more detail, equation 458 changes to

$$b \times g_{\mu\nu} \equiv \left(\frac{1}{4 \times \pi \times D} \times \left(\left(F_{\mu\,\mathrm{c}} \times F_{\nu}{}^{\mathrm{c}}\right) + \left(\frac{1}{4} \times g_{\mu\nu} \times F_{\mathrm{de}} \times F^{\mathrm{de}}\right)\right)\right) \times g_{\mu\nu} \times g^{\mu\nu} \tag{459}$$

or to

$$b \times g_{\mu\nu} \equiv \left(\frac{1}{4 \times \pi} \times \left(\left(\frac{F_{\mu\,\mathrm{c}} \times F_{\nu}{}^{\mathrm{c}}}{D}\right) + \left(\frac{g_{\mu\nu} \times F_{\mathrm{de}} \times F^{\mathrm{de}}}{4 \times D}\right)\right)\right) \times g_{\mu\nu} \times g^{\mu\nu} \tag{460}$$

and to

$$b \times g_{\mu\nu} \equiv \left(\frac{1}{4 \times \pi} \times \left(\left(\frac{F_{\mu\,\mathrm{c}} \times F_{\nu}{}^{\mathrm{c}} \times g^{\mu\nu}}{D}\right) + \left(\frac{g_{\mu\nu} \times g^{\mu\nu} \times F_{\mathrm{de}} \times F^{\mathrm{de}}}{4 \times D}\right)\right)\right) \times g_{\mu\nu} \tag{461}$$

Rearranging equation 461, it is

$$b \times g_{\mu\nu} \equiv \left(\frac{1}{4 \times \pi \times D} \times \left(\left(F_{\mu\,\mathrm{c}} \times F_{\nu}{}^{\mathrm{c}} \times g^{\mu\nu}\right) + \left(\frac{g_{\mu\nu} \times g^{\mu\nu} \times F_{\mathrm{de}} \times F^{\mathrm{de}}}{4}\right)\right)\right) \times g_{\mu\nu} \tag{462}$$

It is $F_{\nu}{}^{\mathrm{c}} \times g^{\mu\nu} \equiv F^{\mu\,\mathrm{c}}$. Additionally, it is $g_{\mu\nu} \times g^{\mu\nu} \equiv D$. Equation 462 becomes

$$b \times g_{\mu\nu} \equiv \left(\frac{1}{4 \times \pi \times D} \times \left(\left(F_{\mu\,\mathrm{c}} \times F^{\mu\,\mathrm{c}}\right) + \left(\frac{D \times F_{\mathrm{de}} \times F^{\mathrm{de}}}{4}\right)\right)\right) \times g_{\mu\nu} \tag{463}$$

□

Theorem 3.23 (The geometrical form of the stress energy tensor of the electromagnetic field ${}_{\mathrm{W}}\mathrm{E}_{\mu\nu}$ II)**.** *In general, conditions are thinkable where geometrical form of the stress energy tensor of the electromagnetic field follows as*

$$b \times g_{\mu\nu} \equiv \left(\frac{(4 + D) \times F_{1}}{4 \times 4 \times \pi \times D}\right) \times g_{\mu\nu} \tag{464}$$

Proof by direct proof. In line with the equation 463, it is

$$b \times g_{\mu\nu} \equiv \left(\frac{1}{4 \times \pi \times D} \times \left(\left(F_{\mu\,\mathrm{c}} \times F^{\mu\,\mathrm{c}}\right) + \left(\frac{D}{4} \times F_{\mathrm{de}} \times F^{\mathrm{de}}\right)\right)\right) \times g_{\mu\nu} \tag{465}$$

Two invariant Lorentz scalars of electromagnetic fields (i.e. electromagnetic invariants) are discussed in literature (Escobar and Urrutia, 2014). The first quadratic Lorentz invariant (see equation 2.63, p. 46), denoted as F_1, is determined as $F_1 \equiv F_{\mathrm{de}} \times F^{\mathrm{de}}$. Among the many references in the literature related to invariant Lorentz scalars, at least to the best of our knowledge, equation 465 becomes

$$b \times g_{\mu\nu} \equiv \left(\frac{1}{4 \times \pi \times D} \times \left(\left(F_{\mu\,\mathrm{c}} \times F^{\mu\,\mathrm{c}}\right) + \left(\frac{D}{4} \times F_{1}\right)\right)\right) \times g_{\mu\nu} \tag{466}$$

Under conditions where $F_1 \equiv F_{\mathrm{de}} \times F^{\mathrm{de}} \equiv F_{\mu\,\mathrm{c}} \times F^{\mu\,\mathrm{c}}$ (see equation 46, p. 15), equation 466 simplifies further and becomes

$$b \times g_{\mu\nu} \equiv \left(\frac{1}{4 \times \pi \times D} \times \left(\left(F_{1}\right) + \left(\frac{D}{4} \times F_{1}\right)\right)\right) \times g_{\mu\nu} \tag{467}$$

or

$$b \times g_{\mu\nu} \equiv \left(\frac{1}{4 \times \pi \times D} \times \left(\left(\frac{4 \times F_{1}}{4}\right) + \left(\frac{D}{4} \times F_{1}\right)\right)\right) \times g_{\mu\nu} \tag{468}$$

or

$$b \times g_{\mu\nu} \equiv \left(\frac{1}{4 \times \pi \times D} \times \left(\left(\frac{4 + D}{4}\right) \times F_{1}\right)\right) \times g_{\mu\nu} \tag{469}$$

or in general

$$b \times g_{\mu\nu} \equiv \left(\frac{(4 + D) \times F_{1}}{4 \times 4 \times \pi \times D}\right) \times g_{\mu\nu} \tag{470}$$

□

Theorem 3.24 (The geometrical form of the stress energy tensor of the electromagnetic field)**.** *Theoretically, it appears to be that there are conditions where the stress-energy tensor of the electromagnetic field takes the form*

$$b \times g_{\mu\nu} \equiv \left(\frac{\hbar \times F_1}{h \times D}\right) \times g_{\mu\nu} \tag{471}$$

Proof by direct proof. In line with the equation 467, it is

$$b \times g_{\mu\nu} \equiv \left(\frac{1}{4 \times \pi \times D} \times \left((F_1) + \left(\frac{D}{4} \times F_1\right)\right)\right) \times g_{\mu\nu} \tag{472}$$

However, the concrete form of stress-energy tensor of the electro-magnetic field (see equation 467) follows under conditions of D=4 space-time dimension of general theory of relativity. Rearranging equation 472, we obtain

$$b \times g_{\mu\nu} \equiv \left(\frac{1}{4 \times \pi \times D} \times \left((F_1) + \left(\frac{4}{4} \times F_1\right)\right)\right) \times g_{\mu\nu} \tag{473}$$

or

$$b \times g_{\mu\nu} \equiv \left(\frac{1}{4 \times \pi \times D} \times ((F_1 + F_1))\right) \times g_{\mu\nu} \tag{474}$$

or

$$b \times g_{\mu\nu} \equiv \left(\frac{1}{4 \times \pi \times D} \times (2 \times F_1)\right) \times g_{\mu\nu} \tag{475}$$

or

$$b \times g_{\mu\nu} \equiv \left(\frac{F_1}{2 \times \pi \times D}\right) \times g_{\mu\nu} \tag{476}$$

In general, it is

$$\frac{1}{2} \equiv \frac{\hbar \times \pi}{h} \tag{477}$$

where $\hbar$ is known as the reduced (Dirac, 1926) Planck constant, as the quantum of angular momentum. Later, Dirac wrote: "In Order that the theory may agree with experiment, we must take $\hbar$ equal to $h/2\pi$, where h is the universal constant that was introduced by Planck, known as Planck's constant. "(see also Dirac, 1947, p. 87). Nevertheless, equation 476 simplifies further. It is

$$b \times g_{\mu\nu} \equiv \left(\frac{\hbar \times \pi \times F_1}{h \times \pi \times D}\right) \times g_{\mu\nu} \tag{478}$$

Theoretically, it appears to be possible to express the stress-energy tensor of the electromagnetic field in the form

$$b \times g_{\mu\nu} \equiv \left(\frac{\hbar \times F_1}{h \times D}\right) \times g_{\mu\nu} \tag{479}$$

□

Theorem 3.25 (The geometrical form of the tensor of ordinary matter ${}_{0}\mathrm{E}_{\mu\nu}$ **.** *The geometric form of the stress-energy tensor of ordinary matter, denoted by ${}_{0}E_{\mu\nu} \equiv {}_{f}a^{2} \times g_{\mu\nu}$, is given as*

$$\begin{aligned} a_{\mu\nu} &\equiv a \times g_{\mu\nu} \\ &\equiv {}_{f}a^{2} \times g_{\mu\nu} \\ &\equiv \left(1 - \frac{v^{2}}{c^{2}}\right) \times \left(\frac{8 \times \pi \times \gamma \times T}{c^{4} \times D}\right) \times g_{\mu\nu} \\ &\equiv \left(1 - \frac{v^{2}}{c^{2}}\right) \times \left(\frac{R}{D} - \left(\frac{R}{2}\right) + (\Lambda)\right) \times g_{\mu\nu} \\ &\equiv {}_{0}E_{\mu\nu} \end{aligned} \tag{480}$$

Proof by direct proof. The premise

$$+1 \equiv +1 \tag{481}$$

is true. In the following, we rearrange the premise. After few steps we obtain (see equation 240)

$$\frac{{}_{0}E_{\mathrm{t}}{}^{2}}{{}_{\mathrm{R}}E_{\mathrm{t}}{}^{2}} \equiv \left(1 - \frac{v^{2}}{c^{2}}\right) \tag{482}$$

According to equation 412, the relationship before (equation 482) becomes

$$\frac{{}_{0}E_{\mathrm{t}}{}^{2}}{x_{5} \times \left(\frac{R}{D} - \left(\frac{R}{2}\right) + (\Lambda)\right)} \equiv \left(1 - \frac{v^{2}}{c^{2}}\right) \tag{483}$$

and changes to

$$\frac{{}_{0}E_{\mathrm{t}}{}^{2}}{x_{5}} \equiv \left(1 - \frac{v^{2}}{c^{2}}\right) \times \left(\frac{R}{D} - \left(\frac{R}{2}\right) + (\Lambda)\right) \tag{484}$$

According to equation 402 it is

$$\frac{{}_{0}E_{\mathrm{t}}{}^{2}}{x_{5}} \times g_{\mu\nu} \equiv {}_{\mathrm{f}}a^{2} \times g_{\mu\nu} \tag{485}$$

and

$$\frac{{}_{0}E_{\mathrm{t}}{}^{2}}{x_{5}} \equiv {}_{\mathrm{f}}a^{2} \tag{486}$$

Equation 484 becomes

$${}_{\mathrm{f}}a^{2} \equiv \left(1 - \frac{v^{2}}{c^{2}}\right) \times \left(\frac{R}{D} - \left(\frac{R}{2}\right) + (\Lambda)\right) \tag{487}$$

In general, the geometric form of the stress-energy tensor of ordinary matter, denoted by ${}_{\mathrm{f}}a^{2} \times g_{\mu\nu}$, is given as (see equation 581)

$${}_{\mathrm{f}}a^{2} \times g_{\mu\nu} \equiv \left(1 - \frac{v^{2}}{c^{2}}\right) \times \left(\frac{8 \times \pi \times \gamma \times T}{c^{4} \times D}\right) \times g_{\mu\nu} \equiv {}_{0}E_{\mu\nu} \tag{488}$$

and equally (see equation 581)

$$ {}_{\mathrm{f}}a^{2} \times g_{\mu\nu} \equiv \left(1-\frac{v^{2}}{c^{2}}\right) \times \left(\frac{R}{D}-\left(\frac{R}{2}\right)+(\Lambda)\right) \times g_{\mu\nu} \equiv {}_{0}E_{\mu\nu} \tag{489} $$

□

3.13. *The geometrical form of the tensor ${}_{0}t_{\mu\nu}$*

Theorem 3.26 (The geometrical form of the tensor ${}_{0}\mathrm{t}_{\mu\nu}$ **.** *The geometrical form of the tensor ${}_{0}t_{\mu\nu}$ denoted by ${}_{f}c^{2} \times g_{\mu\nu} \equiv {}_{0}t_{\mu\nu}$, is given as*

$$ c_{\mu\nu} \equiv c \times g_{\mu\nu} \equiv {}_{f}c^{2} \times g_{\mu\nu} \equiv \left(1-\frac{v^{2}}{c^{2}}\right) \times \left(\left(\frac{R}{2}\right)-(\Lambda)\right) \times g_{\mu\nu} \equiv {}_{0}t_{\mu\nu} \tag{490} $$

In this context, ${}_{f}c^{2}$ should not be mismatched with c, the speed of the light in vacuum.

Proof by direct proof. The premise

$$ +1 \equiv +1 \tag{491} $$

is true. In the following, we rearrange the premise. After few steps we obtain (see equation 265)

$$ \frac{{}_{0}t_{\mathrm{t}}{}^{2}}{{}_{\mathrm{R}}t_{\mathrm{t}}{}^{2}} \equiv \left(1-\frac{v^{2}}{c^{2}}\right) \tag{492} $$

According to equation 406, the relationship before (equation 492) becomes

$$ \frac{{}_{0}t_{\mathrm{t}}{}^{2}}{x_{5} \times \left(\left(\frac{R}{2}\right)-(\Lambda)\right)} \equiv \left(1-\frac{v^{2}}{c^{2}}\right) \tag{493} $$

and changes to

$$ \frac{{}_{0}t_{\mathrm{t}}{}^{2}}{x_{5}} \equiv \left(1-\frac{v^{2}}{c^{2}}\right) \times \left(\left(\frac{R}{2}\right)-(\Lambda)\right) \tag{494} $$

According to equation 403 it is

$$ \frac{{}_{0}t_{\mathrm{t}}{}^{2}}{x_{5}} \times g_{\mu\nu} \equiv {}_{\mathrm{f}}c^{2} \times g_{\mu\nu} \tag{495} $$

and

$$ \frac{{}_{0}t_{\mathrm{t}}{}^{2}}{x_{5}} \equiv {}_{\mathrm{f}}c^{2} \tag{496} $$

Equation 494 becomes

$$ {}_{\mathrm{f}}c^{2} \equiv \left(1-\frac{v^{2}}{c^{2}}\right) \times \left(\left(\frac{R}{2}\right)-(\Lambda)\right) \tag{497} $$

In general, the tensor ${}_{\mathrm{f}}c^2 \times g_{\mu\nu}$ is given as

$${}_{\mathrm{f}}c^2 \times g_{\mu\nu} \equiv \left(1 - \frac{v^2}{c^2}\right) \times \left(\left(\frac{R}{2}\right) - (\Lambda)\right) \times g_{\mu\nu} \equiv {}_{0}t_{\mu\nu} \tag{498}$$

□

3.14. *The geometrical form of the tensor ${}_{w}t_{\mu\nu}$*

Theorem 3.27 (The geometrical form of the tensor ${}_{\mathrm{w}}t_{\mu\nu}$ **.** *The geometrical form of the tensor ${}_{w}t_{\mu\nu}$ denoted by ${}_{f}d^2 \times g_{\mu\nu} \equiv {}_{w}t_{\mu\nu}$, is given as*

$$d_{\mu\nu} \equiv d \times g_{\mu\nu} \equiv {}_{f}d^2 \times g_{\mu\nu} \equiv \left(\frac{v^2}{c^2}\right) \times \left(\left(\frac{R}{2}\right) - (\Lambda)\right) \times g_{\mu\nu} \equiv {}_{w}t_{\mu\nu} \tag{499}$$

Proof by direct proof. The premise

$$+1 \equiv +1 \tag{500}$$

is true. In the following, we rearrange the premise. After few steps we obtain (see equation 271)

$$\frac{{}_{\mathrm{W}}t_{\mathrm{t}}{}^2}{{}_{\mathrm{R}}t_{\mathrm{t}}{}^2} \equiv \left(\frac{v^2}{c^2}\right) \tag{501}$$

According to equation 406, the relationship before (equation 501) becomes

$$\frac{{}_{\mathrm{W}}t_{\mathrm{t}}{}^2}{x_5 \times \left(\left(\frac{R}{2}\right) - (\Lambda)\right)} \equiv \left(\frac{v^2}{c^2}\right) \tag{502}$$

and changes to

$$\frac{{}_{\mathrm{W}}t_{\mathrm{t}}{}^2}{x_5} \equiv \left(\frac{v^2}{c^2}\right) \times \left(\left(\frac{R}{2}\right) - (\Lambda)\right) \tag{503}$$

According to equation 405 it is

$$\frac{{}_{\mathrm{W}}t_{\mathrm{t}}{}^2}{x_5} \times g_{\mu\nu} \equiv {}_{\mathrm{f}}d^2 \times g_{\mu\nu} \tag{504}$$

and

$$\frac{{}_{\mathrm{W}}t_{\mathrm{t}}{}^2}{x_5} \equiv {}_{\mathrm{f}}d^2 \tag{505}$$

Equation 503 becomes

$${}_{\mathrm{f}}d^2 \equiv \left(\frac{v^2}{c^2}\right) \times \left(\left(\frac{R}{2}\right) - (\Lambda)\right) \tag{506}$$

In general, the tensor ${}_{\mathrm{f}}d^2 \times g_{\mu\nu}$ is given as

$${}_{\mathrm{f}}d^2 \times g_{\mu\nu} \equiv \left(\frac{v^2}{c^2}\right) \times \left(\left(\frac{R}{2}\right) - (\Lambda)\right) \times g_{\mu\nu} \equiv {}_{\mathrm{W}}t_{\mu\nu} \tag{507}$$

□

Einstein field equations becomes

$$R_{\mu\nu} - \left(\frac{R}{2} \times g_{\mu\nu}\right) + (\Lambda \times g_{\mu\nu}) \equiv R_{\mu\nu} - {}_{0}t_{\mu\nu} - {}_{\mathrm{w}}t_{\mu\nu} \equiv {}_{0}E_{\mu\nu} - {}_{\mathrm{w}}E_{\mu\nu} \tag{508}$$

From equation 508 follows that

$${}_{\mathrm{w}}E_{\mu\nu} \equiv \left(\frac{R}{2} \times g_{\mu\nu}\right) - {}_{\mathrm{w}}t_{\mu\nu} \equiv {}_{0}t_{\mu\nu} + (\Lambda \times g_{\mu\nu}) \tag{509}$$

3.15. *Ricci tensor geometrized*

In general relativity, it is common to present the Riemann and Ricci tensors by the Christoffel symbols. However, Christoffel symbols are given through the metric tensor itself. Therefore, giving the Ricci tensor while using the metric tensor explicitly, is theoretically possible and necessary.

Theorem 3.28. *The completely geometrized form of the Ricci tensor $R_{\mu\nu}$ is determined by the equation*

$$x_4 \equiv \frac{R}{D} \tag{510}$$

Proof by direct proof. The fundamental principles of what we understand to be sustainable for presenting the Ricci tensor while using the metric tensor explicitly, is the generally not to be rejected condition that

$$(x_4) \times g_{\mu\nu} \equiv R_{\mu\nu} \tag{511}$$

while x_4 might denote any possible scalar of currently unknown nature. It is therefore the task to solve the issue of which nature x_4 could be. We multiply equation 511 through by the inverse metric tensor $\mathrm{g}^{\mu\nu}$. It is

$$(x_4) \times g_{\mu\nu} \times g^{\mu\nu} \equiv R_{\mu\nu} \times g^{\mu\nu} \tag{512}$$

or (see equation 50)

$$x_4 \times D \equiv R \tag{513}$$

The unknown entity x_4, completely independent of its own specific inner structure, is determined by the relationship

$$x_4 \equiv \frac{R}{D} \tag{514}$$

□

3.16. *Ricci tensor completely geometrized*

Theorem 3.29 (Ricci tensor completely geometrized)**.** *The completely geometrized form of the Ricci tensor $R_{\mu\nu}$ is determined by the equation*

$$R_{\mu\nu} \equiv \left(\frac{R}{D}\right) \times g_{\mu\nu} \tag{515}$$

Proof by direct proof. The complete geometrization of the Ricci tensor $\mathrm{R}_{\mu\nu}$ is expressed by the general validity of the relationship (see equation 511)

$$R_{\mu\nu} \equiv (x_4) \times g_{\mu\nu} \tag{516}$$

However, based on equation 514 it is $x_4 \equiv \frac{R}{D}$. Equation 516 changes accordingly. The Ricci tensor $\mathrm{R}_{\mu\nu}$ given completely through the metric tensor $\mathrm{g}_{\mu\nu}$ does not remain before us as the unknown and follows as

$$R_{\mu\nu} \equiv \left(\frac{R}{D}\right) \times g_{\mu\nu} \tag{517}$$

□

Theorem 3.30. *The correction factor x_1 is given as*

$$x_1 \equiv \frac{\left(\frac{R}{D} - \frac{R}{2} + \Lambda\right)}{\left({}_{R}E_{t}{}^{2}\right)} \tag{518}$$

Proof by direct proof. In general, to assure compatibility with Einstein's general relativity, it is

$$x_1 \times \left({}_{\mathrm{R}}E_{\mathrm{t}}{}^{2}\right) \equiv \left(\frac{R}{D} - \frac{R}{2} + \Lambda\right) \equiv {}_{\mathrm{d}}E_{\mathrm{t}}{}^{2} \tag{519}$$

The correction factor is given as

$$x_1 \equiv \frac{\left(\frac{R}{D} - \frac{R}{2} + \Lambda\right)}{\left({}_{\mathrm{R}}E_{\mathrm{t}}{}^{2}\right)} \equiv \frac{{}_{\mathrm{d}}E_{\mathrm{t}}{}^{2}}{{}_{\mathrm{R}}E_{\mathrm{t}}{}^{2}} \tag{520}$$

□

Theorem 3.31. *The correction factor x_2 is given as*

$$x_2 \equiv \frac{\left(\frac{R}{2} - \Lambda\right)}{\left({}_{R}t_{t}{}^{2}\right)} \tag{521}$$

Proof by direct proof. In general, to assure compatibility with Einstein's general relativity, it is

$$x_2 \times \left({}_{\mathrm{R}}t_{\mathrm{t}}{}^{2}\right) \equiv \left(\frac{R}{2} - \Lambda\right) \equiv {}_{\mathrm{d}}t_{\mathrm{t}}{}^{2} \tag{522}$$

The correction factor is given as

$$x_2 \equiv \frac{\left(\frac{R}{2} - \Lambda\right)}{\left({}_{\mathrm{R}}t_{\mathrm{t}}{}^{2}\right)} \equiv \frac{{}_{\mathrm{d}}t_{\mathrm{t}}{}^{2}}{{}_{\mathrm{R}}t_{\mathrm{t}}{}^{2}} \tag{523}$$

□

Theorem 3.32. *The Einstein scalar* $_{d}\,G_{t}\,^{2}$ *is given as,*

$$\left({}_{d}G_{t}{}^{2}\right) \equiv \left(\frac{R}{D} - \frac{R}{2}\right) \tag{524}$$

Proof by direct proof. In general, it is and has to be that

$$\left({}_{\mathrm{d}}G_{\mathrm{t}}{}^{2}\right) \times g_{\mu\nu} \equiv \left(\frac{R}{D} - \frac{R}{2}\right) \times g_{\mu\nu} \equiv G_{\mu\nu} \tag{525}$$

We multiply equation 525 through by the inverse metric tensor $g^{\mu\nu}$. It is

$$\left({}_{\mathrm{d}}G_{\mathrm{t}}{}^{2}\right) \times g_{\mu\nu} \times g^{\mu\nu} \equiv \left(\frac{R}{D} - \frac{R}{2}\right) \times g_{\mu\nu} \times g^{\mu\nu} \tag{526}$$

Equation 526 changes to

$$\left({}_{\mathrm{d}}G_{\mathrm{t}}{}^{2}\right) \times D \equiv \left(\frac{R}{D} - \frac{R}{2}\right) \times D \tag{527}$$

The Einstein scalar $_{\mathrm{d}}\,\mathrm{G}_{\mathrm{t}}\,^{2}$ is given as

$$\left({}_{\mathrm{d}}G_{\mathrm{t}}{}^{2}\right) \equiv \left(\frac{R}{D} - \frac{R}{2}\right) \tag{528}$$

□

Remark 3.2. *In this publication, the notions G and* $_{d}\,G_{t}^{2}$ *are used interchangeably. However, it is clear to us that conditions were*

$$\left({}_{d}G_{t}{}^{2}\right) \equiv \left(\frac{R}{D} - \frac{R}{2}\right) \equiv 0 \tag{529}$$

needed to be investigated in detail. One logical consequence of such a nature given possibility is that

$$\frac{R}{D} \equiv \frac{R}{2} \tag{530}$$

To say it with the painfully loud voice of the infinitely dark and empty, under conditions of D=2 and lower space-time dimensions, there is no mass. In contrast to such a manifold, nowadays, we can find mass or something concrete everywhere around us. That is to say, mass itself must have developed and is developing somehow from the state of a world of 2 or fewer dimensions to today's number of dimensions. The key question is, therefore, does and how does a part of the energy of a 2 dimensional world or manifold (i.e. determined by particles without mass) will pass over into the state of mass. To get to the heart of the issue, is there a wanderer between different worlds, something similar to the Higgs mechanism or the Higgs field (Aad et al., 2012, Englert and Brout, 1964, Higgs, 1964) which is acting as an intermediate between these two worlds (dimensions), and which is able to explain the generation of mass and at the end of locality too? As known, the cosmic microwave background (CMBR) radiation (Penzias and Wilson, 1965) is an electromagnetic radiation which is considered as one of the major confirmations of the Big Bang theory. A proof that the cosmic microwave background radiation is the determining part of a 2 dimensional world bears equally the germ of ***the refutation***

of the Big Bang theory *too. Nonetheless, it is not completely irrelevant to assume that the state of pure* ***non-locality*** *can be found in a world of 2 or fewer dimensions. Under these conditions, a gravitational potential is of lesser importance while the constancy of the speed of light does not appear to remain an absolute frame of reference. In June 1911 Einstein published in the paper Annalen der Physik an important remark: "Das Prinzip von der Konstanz der Lichtgeschwindigkeit gilt ... nicht ... die Lichtgeschwindigkeit im Schwerefelde [ist, author] eine Funktion des Ortes ..."(Einstein, 1911, p. 906) and elaborated on this point again: "Dagegen bin ich der Ansicht, daß das Prinzip der Konstanz der Lichtgeschwindigkeit sich nur insoweit aufrecht erhalten läßt, als man sich auf raum-zeitliche Gebiete von konstantem Gravitationspotential beschränkt. Hier liegt nach meiner Meinung die Grenze der Gültigkeit zwar nicht des Relativitätsprinzips wohl aber des Prinzips der Konstanz der Lichtgeschwindigkeit und damit unserer heutigen Relativitatstheorie."(Einstein, 1912a, p. 1062) Even if all this might be purely hypothetically for the moment until exact research-results are available, the beginning of our world can be found in pure non-locality too and not only in the big bang. Having said this, I believe that the evolution, the degree and the complexity of the self-organization of objective reality goes hand in hand with the number of space-time dimensions. Whether our four-dimensional world is already part of an even higher-dimensional world might again remain provisionally an open question.*

Theorem 3.33. *Under condition where $x_1 = x_2$, the Einstein field equation becomes*

$$\left(\frac{R}{D}-\frac{R}{2}+\Lambda\right)\times g_{\mu\nu}\equiv\left(\frac{({}_{R}E_{t}{}^{2})}{({}_{R}t_{t}{}^{2})}\right)\times\left(\frac{R}{2}-\Lambda\right)\times g_{\mu\nu} \tag{531}$$

Proof by direct proof. It is

$$x_1 \equiv x_2 \tag{532}$$

or

$$\frac{\left(\frac{R}{D}-\frac{R}{2}+\Lambda\right)}{({}_{\mathrm{R}}E_{\mathrm{t}}{}^{2})}\equiv\frac{\left(\frac{R}{2}-\Lambda\right)}{({}_{\mathrm{R}}t_{\mathrm{t}}{}^{2})} \tag{533}$$

and

$$\left(\frac{R}{D}-\frac{R}{2}+\Lambda\right)\times\left({}_{\mathrm{R}}t_{\mathrm{t}}{}^{2}\right)\equiv\left(\frac{R}{2}-\Lambda\right)\times\left({}_{\mathrm{R}}E_{\mathrm{t}}{}^{2}\right) \tag{534}$$

and at the end

$$\left(\frac{R}{D}-\frac{R}{2}+\Lambda\right)\equiv\left(\frac{({}_{\mathrm{R}}E_{\mathrm{t}}{}^{2})}{({}_{\mathrm{R}}t_{\mathrm{t}}{}^{2})}\right)\times\left(\frac{R}{2}-\Lambda\right) \tag{535}$$

Equation 535 is multiplied by the metric tensor $g_{\mu\nu}$. The Einstein field equation becomes

$$\left(\frac{R}{D}-\frac{R}{2}+\Lambda\right)\times g_{\mu\nu}\equiv\left(\frac{({}_{\mathrm{R}}E_{\mathrm{t}}{}^{2})}{({}_{\mathrm{R}}t_{\mathrm{t}}{}^{2})}\right)\times\left(\frac{R}{2}-\Lambda\right)\times g_{\mu\nu} \tag{536}$$

□

3.17. The energy scalar E

The expression on the left side of Einstein field equations represents the curvature of space-time as determined by the metric, while the expression on the right side of Einstein field equations represents the matter–energy content of space-time. Einstein described the (local) space-time curvature as $R_{\mu\nu} - \left(\left(\frac{R}{2}\right) \times g_{\mu\nu}\right) + (\Lambda \times g_{\mu\nu})$. In the same respect, the (local) stress-energy and momentum has been described by Einstein as $\left(\frac{8 \times \pi \times \gamma}{c^4}\right) \times T_{\mu\nu}$. Finally, Einstein has been of the opinion that there are conditions where (local) stress-energy and momentum and (local) space-time curvature are determining each other. In general, Einstein field equations relate (local) space-time curvature with (local) energy and momentum by the equation

$$\underbrace{R_{\mu\nu} - \left(\left(\frac{R}{2}\right) \times g_{\mu\nu}\right) + (\Lambda \times g_{\mu\nu})}_{(local)\ space-time\ curvature} \equiv \underbrace{\left(\frac{4 \times 2 \times \pi \times \gamma}{c^4}\right) \times T_{\mu\nu}}_{(local)\ energy\ and\ momentum} \tag{537}$$

Mathematically, it is necessary to consider circumstances that it is possible to take the trace with respect to the metric of both sides of the Einstein field equations.

Theorem 3.34 (Scalar E)**.** *In general, it is*

$$E \equiv {}_{d}E_{t}{}^{2} \equiv \left(\frac{4 \times 2 \times \pi \times \gamma \times T}{c^4 \times D}\right) \tag{538}$$

Proof by direct proof. In general, the equation

$$\left(\frac{4 \times 2 \times \pi \times \gamma}{c^4}\right) \times T_{\mu\nu} \equiv \left(\frac{4 \times 2 \times \pi \times \gamma}{c^4}\right) \times T_{\mu\nu} \tag{539}$$

is correct. However, we want to express the stress-energy tensor of matter (see equation 539) completely by the metric tensor $g_{\mu\nu}$. In general, it is and has to be that,

$$E \times g_{\mu\nu} \equiv {}_{\mathrm{d}}E_{\mathrm{t}}{}^{2} \times g_{\mu\nu} \equiv \left(\frac{4 \times 2 \times \pi \times \gamma}{c^4}\right) \times T_{\mu\nu} \tag{540}$$

However, equation 540 itself leads straightforward to clear consequences. We multiply equation 540 through by the inverse metric tensor $g^{\mu\nu}$. It is

$$E \times g_{\mu\nu} \times g^{\mu\nu} \equiv {}_{\mathrm{d}}E_{\mathrm{t}}{}^{2} \times g_{\mu\nu} \times g^{\mu\nu} \equiv \left(\frac{4 \times 2 \times \pi \times \gamma}{c^4}\right) \times T_{\mu\nu} \times g^{\mu\nu} \tag{541}$$

or (see definition 2.12, equation 50)

$$E \times D \equiv {}_{\mathrm{d}}E_{\mathrm{t}}{}^{2} \times D \equiv \left(\frac{4 \times 2 \times \pi \times \gamma \times T}{c^4}\right) \tag{542}$$

At the end, the scalar E is determined as

$$E \equiv {}_{\mathrm{d}}E_{\mathrm{t}}{}^{2} \equiv \left(\frac{4 \times 2 \times \pi \times \gamma \times T}{c^4 \times D}\right) \tag{543}$$

□

Remark 3.3. *In this publication, the notions E and ${}_{d}\,E_t{}^{2}$ are used interchangeably. However, ${}_{d}\,E_t{}^{2}$ is not identical with ${}_{R}\,E_t{}^{2}$.*

3.18. Stress-energy tensor completely geometrized

Judged from today's perspective, does the definition of the stress-energy tensor $\left(\frac{4 \times 2 \times \pi \times \gamma \times T}{c^4}\right) \times T_{\mu\nu}$ depend on the metric field $g_{\mu\nu}$ at all? Nonetheless, from the geometrical point of view, it is worth being considered that

> "In view of this geometrization, Einstein considered the role of the stress-energy tensor ... a weak spot of the theory because it is a field devoid of any geometrical significance. "
>
> (see also Goenner, 2004, p. 7)

Theorem 3.35 (Stress-energy tensor completely geometrized)**.** *In general, it is*

$$E_{\mu\nu} \equiv \left(\frac{4 \times 2 \times \pi \times \gamma \times T}{c^4 \times D}\right) \times g_{\mu\nu} \tag{544}$$

Proof by direct proof. Based on equation 542, it is

$$E \equiv {}_{\mathrm{d}}E_{\mathrm{t}}{}^{2} \equiv \left(\frac{4 \times 2 \times \pi \times \gamma \times T}{c^4 \times D}\right) \tag{545}$$

Multiplying by the metric tensor $g_{\mu\nu}$, we obtain

$$E \times g_{\mu\nu} \equiv {}_{\mathrm{d}}E_{\mathrm{t}}{}^{2} \times g_{\mu\nu} \equiv \left(\frac{4 \times 2 \times \pi \times \gamma \times T}{c^4 \times D}\right) \times g_{\mu\nu} \tag{546}$$

According to our definition 2.48, equation 546 changes to

$$E_{\mu\nu} \equiv E \times g_{\mu\nu} \equiv {}_{\mathrm{d}}E_{\mathrm{t}}{}^{2} \times g_{\mu\nu} \equiv \left(\frac{4 \times 2 \times \pi \times \gamma \times T}{c^4 \times D}\right) \times g_{\mu\nu} \tag{547}$$

□

3.19. The scalar a

Theorem 3.36 (The scalar a)**.**

$$
{}_{f}a^{2} \equiv \left(1-\left(\frac{v^{2}}{c^{2}}\right)\right) \times \left(\frac{R}{D}-\frac{R}{2}+\Lambda\right) \tag{548}
$$

Proof by direct proof. Under conditions of Einstein's general theory of relativity (Einstein, 1915, 1916, 1917, 1935, Einstein and de Sitter, 1932), the co-variant stress-energy tensor of ordinary matter, in this context denoted by $a_{\mu\nu}$, is given by the equation

$$
a_{\mu\nu} \equiv {}_{\mathrm{f}}a^{2} \times g_{\mu\nu} \tag{549}
$$

For reason of consistency with Einstein's general theory of relativity, it is

$$
{}_{\mathrm{f}}a^{2} \times g_{\mu\nu} \equiv x_{1} \times \left({}_{0}E_{\mathrm{t}}{}^{2}\right) \times g_{\mu\nu} \tag{550}
$$

The correction factor (see equation 520) has been identified as $x_1 \equiv \frac{\left(\frac{R}{D}-\frac{R}{2}+\Lambda\right)}{\left({}_{\mathrm{R}}E_{\mathrm{t}}{}^{2}\right)}$. Equation 550 becomes,

$$
{}_{\mathrm{f}}a^{2} \times g_{\mu\nu} \equiv \left(\frac{\left({}_{0}E_{\mathrm{t}}{}^{2}\right)}{\left({}_{\mathrm{R}}E_{\mathrm{t}}{}^{2}\right)}\right) \times \left(\frac{R}{D}-\frac{R}{2}+\Lambda\right) \times g_{\mu\nu} \tag{551}
$$

or (see equation 240)

$$
{}_{\mathrm{f}}a^{2} \times g_{\mu\nu} \equiv \left(1-\left(\frac{v^{2}}{c^{2}}\right)\right) \times \left(\frac{R}{D}-\frac{R}{2}+\Lambda\right) \times g_{\mu\nu} \tag{552}
$$

There are conditions, where the scalar '${}_{\mathrm{f}}a^{2}$'is determined by the relationship

$$
{}_{\mathrm{f}}a^{2} \equiv \left(1-\left(\frac{v^{2}}{c^{2}}\right)\right) \times \left(\frac{R}{D}-\frac{R}{2}+\Lambda\right) \tag{553}
$$

□

3.20. The scalar b

Theorem 3.37 (The scalar b)**.** *Under certain circumstances, the scalar ${}_{f}b^{2}$ is determined by the equation*

$$
{}_{f}b^{2} \equiv \left(\frac{v^{2}}{c^{2}}\right) \times \left(\frac{R}{D}-\frac{R}{2}+\Lambda\right) \tag{554}
$$

Proof by direct proof. Under conditions of Einstein's general theory of relativity (Einstein, 1915, 1916, 1917, 1935, Einstein and de Sitter, 1932), co-variant stress-energy tensor of the electromagnetic field, in this context denoted by $b_{\mu\nu}$, described by the equation

$$b_{\mu\nu} \equiv \left(\frac{1}{4\times\pi}\times\left(\left(F_{\mu\,\mathrm{c}}\times F_{\nu}{}^{\mathrm{c}}\right)+\left(\frac{1}{4}\times g_{\mu\nu}\times F_{\mathrm{de}}\times F^{\mathrm{de}}\right)\right)\right) \tag{555}$$

(see Lehmkuhl, 2011, p. 13) is given by the relationship

$${}_{\mathrm{f}}b^{2}\times g_{\mu\nu}\equiv x_{1}\times\left({}_{\mathrm{W}}E_{\mathrm{t}}{}^{2}\right)\times g_{\mu\nu} \tag{556}$$

to. The correction factor (see equation 520) has been identified as $x_1 \equiv \frac{\left(\frac{R}{D}-\frac{R}{2}+\Lambda\right)}{\left({}_{\mathrm{R}}E_{\mathrm{t}}{}^{2}\right)}$. Equation 556 becomes,

$${}_{\mathrm{f}}b^{2}\times g_{\mu\nu}\equiv\left({}_{\mathrm{W}}E_{\mathrm{t}}{}^{2}\right)\times\left(\frac{\left(\frac{R}{D}-\frac{R}{2}+\Lambda\right)}{\left({}_{\mathrm{R}}E_{\mathrm{t}}{}^{2}\right)}\right)\times g_{\mu\nu} \tag{557}$$

which is equivalent with

$${}_{\mathrm{f}}b^{2}\times g_{\mu\nu}\equiv\left(\frac{\left({}_{\mathrm{W}}E_{\mathrm{t}}{}^{2}\right)}{\left({}_{\mathrm{R}}E_{\mathrm{t}}{}^{2}\right)}\right)\times\left(\frac{R}{D}-\frac{R}{2}+\Lambda\right)\times g_{\mu\nu} \tag{558}$$

and with (see equation 254)

$${}_{\mathrm{f}}b^{2}\times g_{\mu\nu}\equiv\left(\frac{v^{2}}{c^{2}}\right)\times\left(\frac{R}{D}-\frac{R}{2}+\Lambda\right)\times g_{\mu\nu} \tag{559}$$

There are conditions, where the scalar ${}_{\mathrm{f}}\mathrm{b}^{2}$ is determined by the relationship

$${}_{\mathrm{f}}b^{2}\equiv\left(\frac{v^{2}}{c^{2}}\right)\times\left(\frac{R}{D}-\frac{R}{2}+\Lambda\right)\equiv\left(\frac{v^{2}}{c^{2}}\right)\times\left(\frac{8\times\pi\times\gamma\times T}{c^{4}\times D}\right) \tag{560}$$

□

Remark 3.4. *In D=2 space-time dimension, the scalar b becomes*

$${}_{f}b^{2}\equiv\left(\frac{v^{2}}{c^{2}}\right)\times\left(\frac{R}{D=2}-\frac{R}{2}+\Lambda\right)\equiv\frac{v^{2}}{c^{2}}\times\Lambda \tag{561}$$

3.21. The scalar c

Theorem 3.38 (The scalar c)**.** *There are conditions, where the scalar '${}_{f}c^{2}$'is determined by the relationship*

$${}_{f}c^{2}\equiv\left(1-\left(\frac{v^{2}}{c^{2}}\right)\right)\times\left(\frac{R}{2}-\Lambda\right) \tag{562}$$

The term ${}_{f}c^{2}$ is different from the speed of the light in vacuum, c.

Proof by direct proof. Under conditions of Einstein's general theory of relativity (Einstein, 1915, 1916, 1917, 1935, Einstein and de Sitter, 1932), the tensor of ordinary time, in this context denoted by $c_{\mu\nu}$, is given by the equation

$$c_{\mu\nu} \equiv {}_{\mathrm{f}}c^2 \times g_{\mu\nu} \tag{563}$$

For reason of consistency with Einstein's general theory of relativity, it is

$${}_{\mathrm{f}}c^2 \times g_{\mu\nu} \equiv x_2 \times \left({}_{0}t_{\mathrm{t}}{}^2\right) \times g_{\mu\nu} \tag{564}$$

The correction factor (see equation 523) has been identified as $x_2 \equiv \frac{\left(\frac{R}{2} - \Lambda\right)}{\left({}_{\mathrm{R}}t_{\mathrm{t}}{}^2\right)}$. Equation 564 becomes,

$${}_{\mathrm{f}}c^2 \times g_{\mu\nu} \equiv \left(\frac{\left({}_{0}t_{\mathrm{t}}{}^2\right)}{\left({}_{\mathrm{R}}t_{\mathrm{t}}{}^2\right)}\right) \times \left(\frac{R}{2} - \Lambda\right) \times g_{\mu\nu} \tag{565}$$

or (see equation 265)

$${}_{\mathrm{f}}c^2 \times g_{\mu\nu} \equiv \left(1 - \left(\frac{v^2}{c^2}\right)\right) \times \left(\frac{R}{2} - \Lambda\right) \times g_{\mu\nu} \tag{566}$$

There are conditions, where the scalar 'c'is determined by the relationship

$${}_{\mathrm{f}}c^2 \equiv \left(1 - \left(\frac{v^2}{c^2}\right)\right) \times \left(\frac{R}{2} - \Lambda\right) \tag{567}$$

□

3.22. *The scalar d*

Theorem 3.39 (The scalar d)**.** *Under certain circumstances, the scalar $_{f}d^2$ is determined by the equation*

$${}_{f}d^2 \equiv \left(\frac{v^2}{c^2}\right) \times \left(\frac{R}{2} - \Lambda\right) \tag{568}$$

Proof by direct proof. Under conditions of Einstein's general theory of relativity (Einstein, 1915, 1916, 1917, 1935, Einstein and de Sitter, 1932), the tensor $d_{\mu\nu}$ is given by the relationship

$${}_{\mathrm{f}}d^2 \times g_{\mu\nu} \equiv x_2 \times \left({}_{\mathrm{W}}t_{\mathrm{t}}{}^2\right) \times g_{\mu\nu} \tag{569}$$

to. The correction factor (see equation 523) has been identified as $x_2 \equiv \frac{\left(\frac{R}{2} - \Lambda\right)}{\left({}_{\mathrm{R}}t_{\mathrm{t}}{}^2\right)}$. Equation 569 becomes,

$${}_{\mathrm{f}}d^2 \times g_{\mu\nu} \equiv \left(\frac{\left({}_{\mathrm{W}}t_{\mathrm{t}}{}^2\right)}{\left({}_{\mathrm{R}}t_{\mathrm{t}}{}^2\right)}\right) \times \left(\frac{R}{2} - \Lambda\right) \times g_{\mu\nu} \tag{570}$$

which is equivalent with (see equation 271)

$$_{\mathrm{f}}d^2 \times g_{\mu\nu} \equiv \left(\frac{v^2}{c^2}\right) \times \left(\frac{R}{2} - \Lambda\right) \times g_{\mu\nu} \tag{571}$$

There are conditions, where the scalar d is determined by the relationship

$$_{\mathrm{f}}d^2 \equiv \left(\frac{v^2}{c^2}\right) \times \left(\frac{R}{2} - \Lambda\right) \tag{572}$$

□

Table 9 provides an overview over the relationships outlined just before.

Table 9. Energy, time and space and the four basic fields of nature

		Curvature		
		YES	NO	
Momentum	YES	$\left(1-\left(\frac{v^2}{c^2}\right)\right) \times \left(\frac{R}{D} - \frac{R}{2} + \Lambda\right) \times g_{\mu\nu}$	$\left(\frac{v^2}{c^2}\right) \times \left(\frac{R}{D} - \frac{R}{2} + \Lambda\right) \times g_{\mu\nu}$	$\frac{8 \times \pi \times \gamma}{c^4 \times D} \times g_{\mu\nu}$
	NO	$\left(1-\left(\frac{v^2}{c^2}\right)\right) \times \left(\frac{R}{2} - \Lambda\right) \times g_{\mu\nu}$	$\left(\frac{v^2}{c^2}\right) \times \left(\frac{R}{2} - \Lambda\right) \times g_{\mu\nu}$	$\left(\frac{R}{2} - \Lambda\right) \times g_{\mu\nu}$
		$_{\mathrm{d}}G_{\mathrm{t}}{}^2 \times g_{\mu\nu}$	$_{\mathrm{d}}\underline{G}_{\mathrm{t}}{}^2 \times g_{\mu\nu}$	$R_{\mu\nu}$

3.23. The scalar foundation of the Einstein field equations

Theorem 3.40 (The scalar foundation of the Einstein field equations)**.** *Above all, it is necessary to extend the geometrization of gravitational force to non-gravitational interactions, in particular, to electromagnetism, in order to achieve something like a geometrical unified field theory. Ultimately, not all are comfortable with the geometrization of physics. Besides of all in order to describe all fundamental interactions by appropriate objects of space-time geometry, it is necessary to work out the foundations of the Einstein field equations. The D dimensional foundation of the Einstein field equation is given by*

$$\frac{R}{D} - \left(\frac{R}{2}\right) + (\Lambda) \equiv \left(\frac{8 \times \pi \times \gamma \times T}{c^4 \times D}\right) \tag{573}$$

Proof by modus ponens. **If** the premise of modus ponens

$$\underbrace{+1 = +1}_{(Premise)} \tag{574}$$

is true, **then** the following conclusion

$$\frac{R}{D} - \left(\frac{R}{2}\right) + (\Lambda) \equiv \left(\frac{8 \times \pi \times \gamma \times T}{c^4 \times D}\right) \tag{575}$$

is also true. The premise $(+1 = +1)$ is true. Multiplying the premise $(+1 = +1)$ by Einstein's stress-energy tensor of general relativity $T_{\mu\nu}$, we obtain

$$(+1) \times \left(\frac{4 \times 2 \times \pi \times \gamma}{c^4}\right) \times T_{\mu\nu} \equiv (+1) \times \left(\frac{4 \times 2 \times \pi \times \gamma}{c^4}\right) \times T_{\mu\nu} \tag{576}$$

or

$$\left(\frac{4 \times 2 \times \pi \times \gamma}{c^4}\right) \times T_{\mu\nu} \equiv \left(\frac{2 \times \pi \times \gamma}{c^4}\right) \times 4 \times T_{\mu\nu} \tag{577}$$

Einstein offered **the principle of general covariance** as the foundation of the theory of general relativity and published the relationship between curvature and momentum in the form of his field equations as $R_{\mu\nu} - \left(\left(\frac{R}{2}\right) \times g_{\mu\nu}\right) + (\Lambda \times g_{\mu\nu}) \equiv \left(\frac{8 \times \pi \times \gamma}{c^4}\right) \times T_{\mu\nu}$ (see definition 2.39, equation 139). Equation 577 changes too

$$R_{\mu\nu} - \left(\left(\frac{R}{2}\right) \times g_{\mu\nu}\right) + (\Lambda \times g_{\mu\nu}) \equiv \left(\frac{8 \times \pi \times \gamma}{c^4}\right) \times T_{\mu\nu} \tag{578}$$

Taking the trace with respect to the metric of both sides of the Einstein field equations one gets

$$R_{\mu\nu} \times g^{\mu\nu} - \left(\left(\frac{R}{2}\right) \times g_{\mu\nu} \times g^{\mu\nu}\right) + (\Lambda \times g_{\mu\nu} \times g^{\mu\nu}) \equiv \left(\frac{8 \times \pi \times \gamma}{c^4}\right) \times T_{\mu\nu} \times g^{\mu\nu} \tag{579}$$

Equation 579 simplifies as

$$R - \left(\left(\frac{R}{2}\right) \times D\right) + (\Lambda \times D) \equiv \left(\frac{8 \times \pi \times \gamma}{c^4}\right) \times T \tag{580}$$

where D is the number of space-time dimensions (see definition 2.12, equation 50).Dividing equation 580 by D, the number of space-time dimensions, simplifies equation 580 further. Thus far, from the epistemological standpoint, the generally valid D dimensional foundation of the Einstein field equations is given by

$$\left(\frac{R}{D} - \left(\frac{R}{2}\right) + (\Lambda)\right) \equiv \left(\frac{8 \times \pi \times \gamma \times T}{c^4 \times D}\right) \tag{581}$$

□

3.24. *Einstein's Weltformel*

Einstein is a Kantian, in the details of his philosophy(see also Weinert, 2005) or at least in the outlines of his physics, or both or none? In the end, it will be necessary that each reader answers this question for himself. Clearly, there are numerous non-contradictory points of contact between Kant and Einstein. However, it is an open secret, that Einstein was demonstrably sceptical towards the heart and the core of Kant's philosophy, Kant's apriorism. Again, Einstein is writing: 'Das ist die Erbsünde Kants, dass Begriffe und Kategorien, die nicht aus der Erfahrung gewonnen werden können, zum Verständnis dieser Erfahrung notwendig sein sollen. Unbefriedigend bleibt dabei aber immer die Willkür der Auswahl derjenigen Elemente, die man als apriorisch bezeichnet.'(see also Fölsing, Albrecht, 1993, p. 544). Translated into broken English: 'That is Kant's fundamental error that concepts and categories which cannot be obtained from experience should be necessary for an understanding of this experience. However, the arbitrary choice of those elements that are called a priori remains unsatisfactory. 'At the end, there appears not to be a better way to sum up the contradiction between Immanuel Kant's theory of knowledge and Einstein's theory of relativity but to cite Einstein himself.

"The elements of ... reality cannot be determined by a priori philosophical considerations,

but must be found by an appeal to results of experiments and measurements. "

(see Einstein et al., 1935b, p. 777)

Through the development of science, philosophers reacted to scientific discoveries by reshaping their notions. Especially, the evolution of physical concepts guided the evolution of philosophical notions like space and time too and vice versa. Considered from the historical point of view, a chain of rejections started at least with Leibniz. Leibniz rejected Newton's concept of absolute space and absolute time, Kant himself rejected Leibnizian relationism with respect to space and time et cetera. The way out, so it seemed to Einstein, which is much closer to Leibniz than to Kant, was to regard space and time as space-time, as a union of space and time. As is documented by publications, Albert Einstein's (1879–1955) own post-Kantianism philosophy of science culminated in the Einstein field equations(see also Einstein, 1916, 1917) derived as $G_{\mu\nu} + \left(\Lambda \times g_{\mu\nu}\right) = \left(\frac{8 \times \pi \times \gamma}{c^4}\right) \times T_{\mu\nu}$ where $G_{\mu\nu}$ is a rank-2 co-variant tensor describing the space-time curvature (the Einstein tensor (see Kasner, 1920)) and $\left(\frac{8 \times \pi \times \gamma}{c^4}\right) \times T_{\mu\nu}$ is the stress–energy tensor or the stress–energy–momentum tensor or the energy–momentum tensor of matter while Λ is the cosmological constant(see also Einstein and Sitter, 1932, Einstein, 1917). Misner et al. (see also Misner et al., 1973, p. 5) summarized the Einstein's geometric theory of gravity as follows:

"**Space** acts on *matter*, telling it how to **move**.

In turn,

matter reacts back on space, telling it how to ***curve***. "

(see also Misner et al., 1973, p. 5)

Theorem 3.41. *Thus far, let $R_{\mu\nu} = a_{\mu\nu} + b_{\mu\nu} + c_{\mu\nu} + d_{\mu\nu}$ denote the second-rank co-variant Ricci curvature tensor, the trace of the Riemann curvature tensor. Let $a_{\mu\nu}$, $b_{\mu\nu}$, $c_{\mu\nu}$ and $d_{\mu\nu}$ denote the* *four basic fields of nature, while $a_{\mu\nu}$ denote the stress-energy tensor of ordinary matter, $b_{\mu\nu}$ denote the stress-energy tensor of the electromagnetic field. As it has been already proven several times, it is $a_{\mu\nu} \equiv a \times g_{\mu\nu}$ and $b_{\mu\nu} \equiv b \times g_{\mu\nu}$ and $c_{\mu\nu} \equiv c \times g_{\mu\nu}$ and $d_{\mu\nu} \equiv d \times g_{\mu\nu}$ and $E_{\mu\nu} = \frac{8 \times \pi \times \gamma \times T}{c^4 \times D} \times g_{\mu\nu} = a_{\mu\nu} + b_{\mu\nu}$ and $G_{\mu\nu} = G \times g_{\mu\nu} = \left(\frac{R}{D} - \frac{R}{2}\right) \times g_{\mu\nu} = a_{\mu\nu} + c_{\mu\nu}$. In general, Einstein's Weltformel is determined as*

$$k = \frac{(a \times d) - (b \times c)}{\sqrt[2]{\left(\left(\frac{R}{D} - \frac{R}{2}\right) \times \left(\frac{R}{2}\right) \times \left(\frac{8 \times \pi \times \gamma \times T}{c^4 \times D}\right) \times \left(\frac{R}{2} - \Lambda\right)\right)}} \tag{582}$$

Proof by direct proof. The premise

$$+1 = +1 \tag{583}$$

is true. The Ricci scalar R follows as $R = R_{\mu\nu} \times g^{\mu\nu} = a_{\mu\nu} \times g^{\mu\nu} + b_{\mu\nu} \times g^{\mu\nu} + c_{\mu\nu} \times g^{\mu\nu} + d_{\mu\nu} \times g^{\mu\nu}$. Once again, it is more than ever necessary to repeat the point that a, b, c and d are only placeholders of the scalars of the four basic fields of nature. These parameters don't say anything about the content or the value of the scalar. Under other conditions these placeholders were denoted as ${}_{\mathrm{f}}\mathrm{a}^2$, ${}_{\mathrm{f}}\mathrm{b}^2$, c^2 and ${}_{\mathrm{f}}\mathrm{d}^2$. This should not engender any confusion. Multiplying equation 583 by $\frac{R}{D} = a + b + c + d$,it is

$$\left(\frac{R}{D}\right) = \left(\frac{R}{D}\right) \tag{584}$$

Multiplying equation 584 by the scalar 'a'of the stress-energy tensor of ordinary matter, $a_{\mu\nu}$, it is

$$\left(\frac{R}{D} \times a\right) = \left(\frac{R}{D} \times a\right) \tag{585}$$

Subtracting the term $\left(\left(\left(\frac{8 \times \pi \times \gamma \times T}{c^4 \times D}\right) \times G\right)\right)$, we obtain

$$\left(\frac{R}{D} \times a\right) - \left(\left(\left(\frac{8 \times \pi \times \gamma \times T}{c^4 \times D}\right) \times G\right)\right) = \left(\frac{R}{D} \times a\right) - \left(\left(\left(\frac{8 \times \pi \times \gamma \times T}{c^4 \times D}\right) \times G\right)\right) \tag{586}$$

It is $\frac{R}{D} = a + b + c + d$ Equation 586 changes to

$$\begin{aligned}\left(\frac{R}{D} \times a\right) - \left(\left(\left(\frac{8 \times \pi \times \gamma \times T}{c^4 \times D}\right) \times G\right)\right) &= (a + b + c + d) \times a \\ &- \left(\left(\left(\frac{8 \times \pi \times \gamma \times T}{c^4 \times D}\right) \times G\right)\right)\end{aligned} \tag{587}$$

Additionally, it is $\left(\frac{8 \times \pi \times \gamma \times T}{c^4 \times D}\right) = a + b$ and $G = a + c$ equation 587 changes to

$$\begin{aligned}\left(\frac{R}{D} \times a\right) - \left(\left(\left(\frac{8 \times \pi \times \gamma \times T}{c^4 \times D}\right) \times G\right)\right) &= (a + b + c + d) \times a \\ &- ((a + b) \times (a + c))\end{aligned} \tag{588}$$

Simplifying equation 588, it is

$$\left(\frac{R}{D}\times a\right)-\left(\left(\left(\left(\frac{8\times\pi\times\gamma\times T}{c^4\times D}\right)\times G\right)\right)\right)=(a\times a)+(a\times b)+(a\times c)+(a\times d)$$
$$-(a\times a)-(a\times b)-(a\times c)-(b\times c) \tag{589}$$

and equally according to our today mathematical rules

$$\left(\frac{R}{D}\times a\right)-\left(\left(\left(\left(\frac{8\times\pi\times\gamma\times T}{c^4\times D}\right)\times G\right)\right)\right)=(a\times d)-(b\times c) \tag{590}$$

Equation 590 is divided by the term $\sqrt[2]{\left(\left(\frac{R}{D}-\frac{R}{2}\right)\times\left(\frac{R}{2}\right)\times\left(\frac{8\times\pi\times\gamma\times T}{c^4\times D}\right)\times\left(\frac{R}{2}-\Lambda\right)\right)}$. Equation 590 changes to

$$\frac{\left(\frac{R}{D}\times a\right)-\left(\left(\left(\left(\frac{8\times\pi\times\gamma\times T}{c^4\times D}\right)\times G\right)\right)\right)}{\sqrt[2]{\left(\left(\frac{R}{D}-\frac{R}{2}\right)\times\left(\frac{R}{2}\right)\times\left(\frac{8\times\pi\times\gamma\times T}{c^4\times D}\right)\times\left(\frac{R}{2}-\Lambda\right)\right)}}=\frac{(a\times d)-(b\times c)}{\sqrt[2]{\left(\left(\frac{R}{D}-\frac{R}{2}\right)\times\left(\frac{R}{2}\right)\times\left(\frac{8\times\pi\times\gamma\times T}{c^4\times D}\right)\times\left(\frac{R}{2}-\Lambda\right)\right)}} \tag{591}$$

In the following, we would like to point out the following definition.

Definition 3.6 (Einstein's Weltformel scalar k).

$$k=\frac{\left(\frac{R}{D}\times a\right)-\left(\left(\left(\left(\frac{8\times\pi\times\gamma\times T}{c^4\times D}\right)\times G\right)\right)\right)}{\sqrt[2]{\left(\left(\frac{R}{D}-\frac{R}{2}\right)\times\left(\frac{R}{2}\right)\times\left(\frac{8\times\pi\times\gamma\times T}{c^4\times D}\right)\times\left(\frac{R}{2}-\Lambda\right)\right)}} \tag{592}$$

Based on definition 3.6, equation 591 changes and another adequate expression of the scalar k of Einstein's Weltformel follows as

$$k=\frac{(a\times d)-(b\times c)}{\sqrt[2]{\left(\left(\frac{R}{D}-\frac{R}{2}\right)\times\left(\frac{R}{2}\right)\times\left(\frac{8\times\pi\times\gamma\times T}{c^4\times D}\right)\times\left(\frac{R}{2}-\Lambda\right)\right)}} \tag{593}$$

Multiplying equation 593 by the metric tensor $g_{\mu\nu\ldots}$, the D dimensional, n-th index Einstein's Weltformel (causal relationship k) is given by

$$k\times g_{\mu\nu\ldots}=k_{\mu\nu\ldots}=\left(\frac{(a\times d)-(b\times c)}{\sqrt[2]{\left(\left(\frac{R}{D}-\frac{R}{2}\right)\times\left(\frac{R}{2}\right)\times\left(\frac{8\times\pi\times\gamma\times T}{c^4\times D}\right)\times\left(\frac{R}{2}-\Lambda\right)\right)}}\right)\times g_{\mu\nu\ldots} \tag{594}$$

while a, b, c, d denote scalars of the four basic fields of nature. Multiplying equation 593 by the wave function Ψ, it is

$$k \times \Psi = \left(\frac{(a \times d) - (b \times c)}{\sqrt[2]{\left(\left(\frac{R}{D} - \frac{R}{2}\right) \times \left(\frac{R}{2}\right) \times \left(\frac{8 \times \pi \times \gamma \times T}{c^4 \times D}\right) \times \left(\frac{R}{2} - \Lambda\right)\right)}} \right) \times \Psi \tag{595}$$

Multiplying equation 593 by the wave function Ψ and the metric tensor $g_{\mu\nu}$, it is

$$k \times \Psi \times g_{\mu\nu\ldots} = \left(\frac{(a \times d) - (b \times c)}{\sqrt[2]{\left(\left(\frac{R}{D} - \frac{R}{2}\right) \times \left(\frac{R}{2}\right) \times \left(\frac{8 \times \pi \times \gamma \times T}{c^4 \times D}\right) \times \left(\frac{R}{2} - \Lambda\right)\right)}} \right) \times \Psi \times g_{\mu\nu\ldots} \tag{596}$$

However, equation 596 does not exclude conditions where $k \equiv \Psi$. Nonetheless, equation 596 changes (according to definition 2.3) to

$$k \times \Psi_{\mu\nu\ldots} = \left(\frac{(a \times d) - (b \times c)}{\sqrt[2]{\left(\left(\frac{R}{D} - \frac{R}{2}\right) \times \left(\frac{R}{2}\right) \times \left(\frac{8 \times \pi \times \gamma \times T}{c^4 \times D}\right) \times \left(\frac{R}{2} - \Lambda\right)\right)}} \right) \times \Psi_{\mu\nu\ldots} \tag{597}$$

□

Remark 3.5. *It is an open question whether the relationship*

$$k^{+2} = \frac{((a \times d) - (b \times c)) \times ((a \times d) - (b \times c))}{\left(\left(\frac{R}{D} - \frac{R}{2}\right) \times \left(\frac{R}{2}\right) \times \left(\frac{8 \times \pi \times \gamma \times T}{c^4 \times D}\right) \times \left(\frac{R}{2} - \Lambda\right)\right)} \equiv \frac{\chi^2}{\frac{R}{D}} \tag{598}$$

is valid where χ is chi square(see also Pearson, 1904, p. 6) distributed. Depending upon several factors, equation 593 derived as

$$k = \frac{(a \times d) - (b \times c)}{\sqrt[2]{\left(\left(\frac{R}{D} - \frac{R}{2}\right) \times \left(\frac{R}{2}\right) \times \left(\frac{8 \times \pi \times \gamma \times T}{c^4 \times D}\right) \times \left(\frac{R}{2} - \Lambda\right)\right)}} \tag{599}$$

can be equal to zero, less than zero or greater than zero. Does equation 599 allow something like a ***creatio ex nihilio****, a creation or a beginning of our world out of nothing, however we may define nothing? Are questions like these beyond any human experience?*

Physics and the Einstein field equations many times study only something what exists. Consequently, one might expect physics including the Einstein field equations to have little to say about the special *case in which nothing, an absence of something, exists. It is necessary to point out,* ***nothing exists****, it is a nothing, but it is given too. As far as simplicity is concerned, there is a tie between the Einstein field equations and Nothingness as such.*

Let us imagine a manifold where $a_{\mu\nu}$*, the stress energy tensor of ordinary matter vanish, where* $a_{\mu\nu}$ *= 0. Equation 590 changes to*

$$\left(R_{\mu\nu} \cap 0_{\mu\nu}\right) - \left(\left(\left(\frac{8 \times \pi \times \gamma}{c^4}\right) \times T_{\mu\nu}\right) \cap G_{\mu\nu}\right) = \left(0_{\mu\nu} \cap d_{\mu\nu}\right) - \left(b_{\mu\nu} \cap c_{\mu\nu}\right) \tag{600}$$

But by the same reasoning there is nothing rather than something or

$$-\left(\left(\left(\frac{8 \times \pi \times \gamma}{c^4}\right) \times T_{\mu\nu}\right) \cap G_{\mu\nu}\right) = -\left(b_{\mu\nu} \cap c_{\mu\nu}\right) \tag{601}$$

while $b_{\mu\nu}$ *contains the entire stress-energy of the manifold studied while* $c_{\mu\nu}$ *contains the entire gravitational field. Reissner–Nordström were the first to describe a kind of an electrovacuum, a geometry around a charged spherical mass (see also Nordström, 1918, Reissner, 1916). However, in contrast to Reissner–Nordström electrovacuum, equation 601 describes regions without ordinary mass (*$a_{\mu\nu}$ *= 0) where the stress energy tensor* $\left(\frac{8 \times \pi \times \gamma}{c^4}\right) \times T_{\mu\nu}$ *has not vanished but has passed over into the stress-energy tensor of the electromagnetic field,* $b_{\mu\nu}$*, completely. In other words, it is* $\left(\frac{8 \times \pi \times \gamma}{c^4}\right) \times T_{\mu\nu} = b_{\mu\nu}$*. Under these conditions, the stress-energy tensor of the electromagnetic field is the source of the gravitational field (*$c_{\mu\nu}$*), which itself has not vanished either. Equation 601 describes the flat, the empty negative and is more or less one typical feature of the exact electro vacuum solution of the Einstein field equation and may be of any emptiness at all.*

The question is justified, is emptiness or nothingness absolute or relative or both or none and can we know self-consciously anything at all about any emptiness, the void, which does not exist? Radical advocates of a creation out of nothing prefer the possibility of total nothingness. Equation 601 in turn implies that there can be some nothingness but the same is relative too. However, a beginning of our world out of the empty negative (see equation 601) is possible, theoretically.

3.25. Ricci scalar, lambda and anti-lambda

Theorem 3.42 (Ricci scalar, lambda and anti-lambda)**.** *The logical need and the justification for a concept of lambda and anti-lambda as the most basic foundation of nature has been presented by myself to the public on Thursday, 13th of June 2013 about 15.35-15.55 local time at the conference Quantum Theory: Advances and Problems - (QTAP) in Växjö, Sweden, June 10-13,2013 and has been published (Barukčić, 2015a) later. In general, it is*

$$\left(R \times g_{\mu\nu}\right) + 0 \equiv \left(\underline{\Lambda} \times g_{\mu\nu}\right) + \left(\Lambda \times g_{\mu\nu}\right) \tag{602}$$

Proof by direct proof. **If** the premise

$$\underbrace{+1 = +1}_{(Premise)} \tag{603}$$

is true, **then** the following conclusion

$$R \times g_{\mu\nu} + 0 \equiv \left(\underline{\Lambda} \times g_{\mu\nu}\right) + \left(\Lambda \times g_{\mu\nu}\right) \tag{604}$$

is also true, again the absence of any technical errors presupposed. The premise

$$+1 \equiv +1 \tag{605}$$

is true. We multiply equation 605 through by $R \times g_{\mu\nu}$, it is

$$R \times g_{\mu\nu} \equiv R \times g_{\mu\nu} \tag{606}$$

Adding zero, we obtain

$$\left(R \times g_{\mu\nu}\right) + 0 \equiv \left(R \times g_{\mu\nu}\right) - \left(\Lambda \times g_{\mu\nu}\right) + \left(\Lambda \times g_{\mu\nu}\right) \tag{607}$$

In accordance with our definition (see definition 2.44), it is $\left(\underline{\Lambda} \times g_{\mu\nu}\right) \equiv \left(R \times g_{\mu\nu}\right) - \left(\Lambda \times g_{\mu\nu}\right)$. Equation 607 changes to

$$\left(R \times g_{\mu\nu}\right) + 0 \equiv \left(\underline{\Lambda} \times g_{\mu\nu}\right) + \left(\Lambda \times g_{\mu\nu}\right) \tag{608}$$

□

3.26. Nature as the unity and the struggle of opposites

Theorem 3.43 (Nature as the unity and the struggle of opposites)**.** *The Einstein field equations describe the unity and the struggle of opposites as*

$$\left(\left(\frac{1}{D} + \left(\frac{1}{2}\right)\right) \times \Lambda \times g_{\mu\nu}\right) + \left(\left(\frac{1}{D} - \left(\frac{1}{2}\right)\right) \times \underline{\Lambda} \times g_{\mu\nu}\right) \equiv \left(\frac{8 \times \pi \times \gamma \times T}{c^4 \times D}\right) \times g_{\mu\nu} \tag{609}$$

Proof by direct proof. **If** the premise

$$\underbrace{+1 = +1}_{(Premise)} \tag{610}$$

is true, **then** the following conclusion

$$\left(\left(\frac{1}{D}+\left(\frac{1}{2}\right)\right)\times\Lambda\times g_{\mu\nu}\right)+\left(\left(\frac{1}{D}-\left(\frac{1}{2}\right)\right)\times\underline{\Lambda}\times g_{\mu\nu}\right)\equiv\left(\frac{8\times\pi\times\gamma\times T}{c^4\times D}\right)\times g_{\mu\nu} \tag{611}$$

is also true, again the absence of any technical errors presupposed. The premise

$$+1\equiv+1 \tag{612}$$

is true. We multiply equation 612 through by equation 581 . It is

$$\frac{R}{D}-\left(\frac{R}{2}\right)+(\Lambda)\equiv\left(\frac{8\times\pi\times\gamma\times T}{c^4\times D}\right) \tag{613}$$

According to equation 608, it is $R\equiv(\underline{\Lambda})+(\Lambda)$. Equation 613 simplifies as

$$\frac{\Lambda+\underline{\Lambda}}{D}-\left(\frac{\Lambda+\underline{\Lambda}}{2}\right)+(\Lambda)\equiv\left(\frac{8\times\pi\times\gamma\times T}{c^4\times D}\right) \tag{614}$$

Equation 614 is identical with

$$\frac{\Lambda}{D}+\frac{\underline{\Lambda}}{D}-\left(\frac{\Lambda}{2}\right)-\left(\frac{\underline{\Lambda}}{2}\right)+(\Lambda)\equiv\left(\frac{8\times\pi\times\gamma\times T}{c^4\times D}\right) \tag{615}$$

and with

$$(\Lambda)+\frac{\Lambda}{D}-\left(\frac{\Lambda}{2}\right)+\frac{\underline{\Lambda}}{D}-\left(\frac{\underline{\Lambda}}{2}\right)\equiv\left(\frac{8\times\pi\times\gamma\times T}{c^4\times D}\right) \tag{616}$$

and finally with

$$\left(\frac{1}{D}+1-\left(\frac{1}{2}\right)\right)\times\Lambda+\left(\frac{1}{D}-\left(\frac{1}{2}\right)\right)\times\underline{\Lambda}\equiv\left(\frac{8\times\pi\times\gamma\times T}{c^4\times D}\right) \tag{617}$$

The unity and the struggle of opposites as the foundation of nature is equally the foundation of the Einstein field equations as

$$\left(\left(\frac{1}{D}+\left(\frac{1}{2}\right)\right)\times\Lambda\times g_{\mu\nu}\right)+\left(\left(\frac{1}{D}-\left(\frac{1}{2}\right)\right)\times\underline{\Lambda}\times g_{\mu\nu}\right)\equiv\left(\frac{8\times\pi\times\gamma\times T}{c^4\times D}\right)\times g_{\mu\nu} \tag{618}$$

□

In a very special way, it is necessary to drew readers attention to the grave consequences which theoretically might be given under conditions where $\left(\frac{8\times\pi\times\gamma\times T}{c^4\times D}\right)\times g_{\mu\nu}\equiv+0$. Under these circumstances, equation 618 changes to

$$\mathbf{+}\left(\left(\frac{1}{D}+\left(\frac{1}{2}\right)\right)\times\Lambda\times g_{\mu\nu}\right)\equiv\mathbf{-}\left(\left(\frac{1}{D}-\left(\frac{1}{2}\right)\right)\times\underline{\Lambda}\times g_{\mu\nu}\right) \tag{619}$$

the state of pure symmetry, the unity and the struggle between a purely positive and a purely negative.

The Schrödinger equation (Schrödinger, 1926) is not compatible with general and special theory of relativity to a necessary extent. It is therefore hardly surprising that there are already several trials to formulate relativistic versions (Bel and Ruiz, 1998) of the Schrödinger equation to ensure compatibility with relativity theory, the Klein–Gordon–Fock equation (Gordon, 1926, Klein, 1926) or the Dirac equation (Dirac, 1928) are some of these attempts. However, the most of the relativistic wave equations known are more or less a quantized version of the relativistic energy–momentum relation. Nonetheless, a relativistic Schrödinger wave equation (Barukčić, 2013) incorporating the gravitational field too is everywhere as far as the eye can reach, neither known nor generally accepted.

Theorem 3.44 (Relativistic Schrödinger equation)**.** *The D dimensional gravitational relativistic Schrödinger equation is given by*

$$\left(\frac{R}{D}-\left(\frac{R}{2}\right)+(\Lambda)\right)\times\Psi\equiv\left(\frac{4\times2\times\pi\times\gamma\times T}{c^4\times D}\right)\times\Psi \tag{620}$$

Proof by direct proof. **If** the premise

$$\underbrace{+1=+1}_{(Premise)} \tag{621}$$

is true, **then** the following conclusion The D dimensional gravitational relativistic Schrödinger equation is given by

$$\left(\frac{R}{D}-\left(\frac{R}{2}\right)+(\Lambda)\right)\times\Psi\equiv\left(\left(\frac{8\times\pi\times\gamma\times T}{c^4\times D}\right)\right)\times\Psi \tag{622}$$

where Ψ is the wave function, is also true, again the absence of any technical errors presupposed. The premise

$$+1\equiv+1 \tag{623}$$

is true. We multiply equation 623 through by $\left(\frac{8\times\pi\times\gamma\times T}{c^4\times D}\right)$. It is

$$\left(\frac{8\times\pi\times\gamma\times T}{c^4\times D}\right)\equiv\left(\frac{4\times2\times\pi\times\gamma\times T}{c^4\times D}\right) \tag{624}$$

Equation 624 changes (in accordance with equation 573) too

$$\left(\frac{R}{D}-\left(\frac{R}{2}\right)+(\Lambda)\right)\equiv\left(\frac{4\times2\times\pi\times\gamma\times T}{c^4\times D}\right) \tag{625}$$

We multiply equation 625 through by the wave function Ψ. The D dimensional gravitational relativistic Schrödinger equation is given by

$$\left(\frac{R}{D}-\left(\frac{R}{2}\right)+(\Lambda)\right)\times\Psi\equiv\left(\frac{4\times2\times\pi\times\gamma\times T}{c^4\times D}\right)\times\Psi \tag{626}$$

Recall that h is Planck's constant (Planck, 1901), $\hbar$ is Dirac's constant (Dirac, 1926), the reduced Planck's constant, and π denote Archimedes constant. Therefore, it is

$$2 \times \pi \equiv \frac{h}{\hbar} \tag{627}$$

Equation 626 changes slightly. The quantized form of the D dimensional gravitational relativistic Schrödinger equation is given by

$$\left(\frac{R}{D} + (\Lambda) - \left(\frac{\pi \times \hbar \times R}{h}\right)\right) \times \Psi \equiv \left(\frac{h \times 4 \times \gamma \times T}{\hbar \times c^4 \times D}\right) \times \Psi \tag{628}$$

□

3.28. *Relativistic Schrödinger equation as the unity of opposites*

Theorem 3.45 (Relativistic Schrödinger equation as the unity of opposites)**.**

$$\left(\left(\frac{1}{D} + \left(\frac{1}{2}\right)\right) \times \Lambda \times \Psi\right) + \left(\left(\frac{1}{D} - \left(\frac{1}{2}\right)\right) \times \underline{\Lambda} \times \Psi\right) \equiv \left(\frac{8 \times \pi \times \gamma \times T}{c^4 \times D}\right) \times \Psi \tag{629}$$

Proof by direct proof. **If** the premise

$$\underbrace{+1 = +1}_{(Premise)} \tag{630}$$

is true, **then** the following conclusion

$$\left(\left(\frac{1}{D} + \left(\frac{1}{2}\right)\right) \times \Lambda \times \Psi\right) + \left(\left(\frac{1}{D} - \left(\frac{1}{2}\right)\right) \times \underline{\Lambda} \times \Psi\right) \equiv \left(\frac{8 \times \pi \times \gamma \times T}{c^4 \times D}\right) \times \Psi \tag{631}$$

is also true, again the absence of any technical errors presupposed. The premise

$$+1 \equiv +1 \tag{632}$$

is true. We multiply equation 632 through by equation 617, it is

$$\left(\left(\frac{1}{D} + \left(\frac{1}{2}\right)\right) \times \Lambda\right) + \left(\left(\frac{1}{D} - \left(\frac{1}{2}\right)\right) \times \underline{\Lambda}\right) \equiv \left(\frac{8 \times \pi \times \gamma \times T}{c^4 \times D}\right) \tag{633}$$

In the Schrödinger picture, let Ψ denote the wave function. The D-dimensional relativistic Schrödinger equation of general relativity follows as

$$\left(\left(\frac{1}{D} + \left(\frac{1}{2}\right)\right) \times \Lambda \times \Psi\right) + \left(\left(\frac{1}{D} - \left(\frac{1}{2}\right)\right) \times \underline{\Lambda} \times \Psi\right) \equiv \left(\frac{8 \times \pi \times \gamma \times T}{c^4 \times D}\right) \times \Psi \tag{634}$$

Let h denote Planck's constant (Planck, 1901), let $\hbar$ denote Dirac's constant (Dirac, 1926), the reduced Planck's constant, let π denote Archimedes constant. It is

$$\frac{1}{2} \equiv \frac{\pi \times \hbar}{h} \tag{635}$$

Equation 634 changes to

$$\left(\left(\frac{1}{D}+\left(\frac{\pi \times \hbar}{h}\right)\right) \times \Lambda \times \Psi\right)+\left(\left(\frac{1}{D}-\left(\frac{\pi \times \hbar}{h}\right)\right) \times \underline{\Lambda} \times \Psi\right) \equiv\left(\left(\frac{h}{\hbar}\right) \times \frac{4 \times \gamma \times T}{D \times c^{4}}\right) \times \Psi \tag{636}$$

□

Remark 3.6. *Einstein's field equations of general theory of relativity under conditions of D=4 space-time dimensions become*

$$\left(\left(\frac{1}{4}+\left(\frac{\pi \times \hbar}{h}\right)\right) \times \Lambda \times \Psi\right)+\left(\left(\frac{1}{4}-\left(\frac{\pi \times \hbar}{h}\right)\right) \times \underline{\Lambda} \times \Psi\right) \equiv\left(\left(\frac{h}{\hbar}\right) \times \frac{\gamma \times T}{c^{4}}\right) \times \Psi \tag{637}$$

3.29. Relativistic wave equation and strings

A theory of quantum gravity incorporating both the principles of quantum theory and general relativity could be able to provide a satisfactory mathematical description of the microstructure of space-time. Put as simply as possible, the gravitational field itself or the curvature of spacetime determined by matter and energy need to be quantized as derived by equation 636. The string theory tried in this context to replace the point particles (photons, electrons, etc) of quantum field theory by one-dimensional extended objects called strings. However, the Kaluza–Klein theory (Kaluza, 1921) is already a kind of historical precursor of string theory and equally a classical unified field theory of gravitation and electromagnetism built around fifth dimension which used a similar idea of Gunnar Nordström. Nordström suggested: "Es wird gezeigt, daß eine einheitliche Behandlung des elektromagnetischen Feldes und des Gravitationsfeldes möglich ist, wenn man die vierdimensionale Raumzeitwelt als eine durch eine fünfdiminsionale Welt gelegte Fläche auffaßt. " (Nordström, 1914). In M-theory (see Witten, 1998, p. 1129) space-time is 11-dimensional (ten spatial dimensions, and one time dimension), while in super-string theory (de Haro et al., 2013) space-time is ten-dimensional (nine spatial dimensions, and one time dimension), and in bosonic string theory (de Haro et al., 2013), it is 26-dimensional. In the year 1995, Edward Witten (see Witten, 1995) suggested that the five consistent versions of superstring theory (type I, type IIA, type IIB, and two versions of heterotic string theory) were just special limiting cases of an eleven-dimensional theory called M-theory. Insofar as issues of string theory and Einstein's general theory of relativity are involved, it should be noted that the Einstein's field equations of the general theory of relativity does not in any way privilege a particular space-time geometry or a space-time dimension. The Einstein's field equations of the general theory of relativity derived as

$$\underbrace{\left(\frac{R}{D} \times g_{\mu\nu\ldots}\right)-\left(\left(\frac{R}{2}\right) \times g_{\mu\nu\ldots}\right)+\left(\Lambda \times g_{\mu\nu\ldots}\right)}_{the\ left-hand\ side} \equiv \underbrace{\left(\frac{4 \times 2 \times \pi \times \gamma \times T}{c^{4} \times D}\right) \times g_{\mu\nu\ldots}}_{the\ right-hand\ side} \tag{638}$$

are able to cope with any space-time dimension.

Theorem 3.46 (Relativistic wave equation and strings)**.** *Under conditions of D = 1 space-time dimensions, the relativistic wave equation is given by*

$$\left(\left(\frac{3}{2}\right)\times\Lambda\times\Psi\right)+\left(\left(\frac{1}{2}\right)\times\underline{\Lambda}\times\Psi\right)\equiv\left(\frac{8\times\pi\times\gamma\times T}{c^{4}}\right)\times\Psi \tag{639}$$

Proof by direct proof. **If** the premise

$$\underbrace{+1=+1}_{(Premise)} \tag{640}$$

is true, **then** the following conclusion

$$\left(\left(\frac{3}{2}\right)\times\Lambda\times\Psi\right)+\left(\left(\frac{1}{2}\right)\times\underline{\Lambda}\times\Psi\right)\equiv\left(\frac{8\times\pi\times\gamma\times T}{c^{4}}\right)\times\Psi \tag{641}$$

is also true, again the absence of any technical errors presupposed. The premise

$$+1\equiv+1 \tag{642}$$

is true. We multiply equation 641 through by equation 634. It is

$$\left(\left(\frac{1}{D}+\left(\frac{1}{2}\right)\right)\times\Lambda\times\Psi\right)+\left(\left(\frac{1}{D}-\left(\frac{1}{2}\right)\right)\times\underline{\Lambda}\times\Psi\right)\equiv\left(\frac{8\times\pi\times\gamma\times T}{c^{4}\times D}\right)\times\Psi \tag{643}$$

Under conditions of D=1 space-time dimension equation 643 becomes

$$\left(\left(\frac{1}{1}+\left(\frac{1}{2}\right)\right)\times\Lambda\times\Psi\right)+\left(\left(\frac{1}{1}-\left(\frac{1}{2}\right)\right)\times\underline{\Lambda}\times\Psi\right)\equiv\left(\frac{4\times2\times\pi\times\gamma\times T}{c^{4}\times1}\right)\times\Psi \tag{644}$$

The relativistic wave equation, which should be able to describe strings to, becomes

$$\left(\left(\frac{3}{2}\right)\times\Lambda\times\Psi\right)+\left(\left(\frac{1}{2}\right)\times\underline{\Lambda}\times\Psi\right)\equiv\left(\left(\frac{h}{\hbar}\right)\times\frac{4\times\gamma\times T}{c^{4}}\right)\times\Psi \tag{645}$$

□

3.30. *The field equations of gravitational waves*

Theorem 3.47 (The field equations of gravitational waves)**.** *In general, it is*

$$R_{\mu\nu}-\left(\left(\left(\frac{R}{2}\right)-(\Lambda)\right)\times\underline{_{gw}g_{\mu\nu}}\right)-\left(\left(\left(\frac{R}{2}\right)-(\Lambda)\right)\times{_{gw}g_{\mu\nu}}\right)\equiv\left(\left(\frac{8\times\pi\times\gamma\times T}{c^{4}\times D}\right)\times g_{\mu\nu}\right) \tag{646}$$

Proof by direct proof. It is

$$\left({}_{\mathrm{gw}}g_{\mu\nu}\right) + \left({}_{\underline{\mathrm{gw}}}g_{\mu\nu}\right) \equiv g_{\mu\nu} \tag{647}$$

Multiplying equation 647 by $\left(\left(\frac{R}{2}\right) - (\Lambda)\right)$, it is

$$\left(\left(\left(\frac{R}{2}\right) - (\Lambda)\right) \times {}_{\mathrm{gw}}g_{\mu\nu}\right) + \left(\left(\left(\frac{R}{2}\right) - (\Lambda)\right) \times {}_{\underline{\mathrm{gw}}}g_{\mu\nu}\right) \equiv \left(\left(\frac{R}{2}\right) - (\Lambda)\right) \times g_{\mu\nu} \tag{648}$$

which is equivalent with

$$\left(\left(\left(\frac{R}{2}\right) - (\Lambda)\right) \times {}_{\mathrm{gw}}g_{\mu\nu}\right) + \left(\left(\left(\frac{R}{2}\right) - (\Lambda)\right) \times {}_{\underline{\mathrm{gw}}}g_{\mu\nu}\right) \equiv R_{\mu\nu} - \left(\left(\frac{8 \times \pi \times \gamma \times T}{c^4 \times D}\right) \times g_{\mu\nu}\right) \tag{649}$$

Einstein field equations in terms of gravitational waves becomes

$$R_{\mu\nu} - \left(\left(\left(\frac{R}{2}\right) - (\Lambda)\right) \times {}_{\underline{\mathrm{gw}}}g_{\mu\nu}\right) - \left(\left(\left(\frac{R}{2}\right) - (\Lambda)\right) \times {}_{\mathrm{gw}}g_{\mu\nu}\right) \equiv \left(\left(\frac{8 \times \pi \times \gamma \times T}{c^4 \times D}\right) \times g_{\mu\nu}\right) \tag{650}$$

□

3.31. The tensor specific metric tensors

Today's majority of physicist are thoroughly convinced that the four fundamental forces of nature which determine everything that happens in the universe are gravity, the weak force, electromagnetism and the strong force. The electroweak theory (Glashow, 1959, Weinberg, 1967) which unified the electromagnetic force with the weak force to form the concept of the electroweak force is compatible with the assumption of the Higgs (Aad et al., 2012, Englert and Brout, 1964, Higgs, 1964) boson. However, a unification of the fundamental forces under a single, unified theory has not been crown with success yet. An outstanding question inevitably comes to mind. Is another point of view on the four fundamental forces of nature logically possible and necessary?

In this context, reference must be made in particular to the necessity to unify the strong force with the weak force into the ordinary force. This procedure moves us one step further towards our goal, which is to ensure the unification of all four fundamental forces of nature. Furthermore, in this publication, the gravitational waves (Einstein, 1918a, LIGO et al., 2016) are treated as an own basic force of nature. Now all the individual pieces will fit together as presented by the table 10.

Table 10. The four basic fields of nature

		Curvature YES	Curvature NO	
Momentum	YES	**Ordinary matter**	Electromagnetic field	Stress-energy tensor
	NO	Gravitational field	Gravitational waves	Anti stress-energy tensor
		Einstein tensor	Anti Einstein tensor	Ricci tensor

Theorem 3.48 (The tensor specific metric tensors)**.** *Let $a_{\mu\nu}$, $b_{\mu\nu}$, $c_{\mu\nu}$ and $d_{\mu\nu}$ denote the four basic fields of nature where $a_{\mu\nu}$ is the stress-energy tensor of ordinary matter with its own metric ${}_{a}g_{\mu\nu}$, $b_{\mu\nu}$ is the stress-energy tensor of the electromagnetic field with its own metric ${}_{b}g_{\mu\nu}$, $c_{\mu\nu}$ is the tensor of the gravitational field with its own metric ${}_{c}g_{\mu\nu}$ and $d_{\mu\nu}$ is the tensor of gravitational waves with its own metric ${}_{d}g_{\mu\nu}$. The tensor specific metric tensor (see table 11) is given by*

$$ {}_{x_4}g_{\mu\nu} \equiv \frac{x_4 \times D}{R} \times g_{\mu\nu} \tag{651}$$

Proof by direct proof. **If** the premise

$$\underbrace{+1 = +1}_{(Premise)} \tag{652}$$

is true, **then** the following conclusion

$$ {}_{\mathrm{x}_4}g_{\mu\nu} \equiv \frac{x_4 \times D}{R} \times g_{\mu\nu} \tag{653}$$

is also true, again the absence of any technical errors presupposed. Our assumption is that

$$\frac{R}{D} \times {}_{\mathrm{x}_4}g_{\mu\nu} \equiv x_4 \times g_{\mu\nu} \tag{654}$$

while ${}_{\mathrm{x}_4}g_{\mu\nu}$ is the specific metric tensor, $g_{\mu\nu}$ is the tensor of general relativity, R is the Ricci scalar, D is the space-time dimension and x_4 is a tensor specific scalar. In general, a tensor specific metric tensor (see table 11) is given by

$$ {}_{\mathrm{x}_4}g_{\mu\nu} \equiv \frac{x_4 \times D}{R} \times g_{\mu\nu} \tag{655}$$

□

Example.

Set x_4 = a for the stress energy tensor of ordinary matter. In this case, it is

$$_{a}g_{\mu\nu} \equiv \frac{a \times D}{R} \times g_{\mu\nu} \tag{656}$$

Table 11. The four basic fields of nature II

		Curvature YES	Curvature NO	
Momentum	YES	$(\frac{R}{D} \times {}_{a}g_{\mu\nu})$	$(\frac{R}{D} \times {}_{b}g_{\mu\nu})$	$(\frac{R}{D} \times {}_{E}g_{\mu\nu})$
	NO	$(\frac{R}{D} \times {}_{c}g_{\mu\nu})$	$(\frac{R}{D} \times {}_{d}g_{\mu\nu})$	$(\frac{R}{D} \times {}_{\underline{E}}g_{\mu\nu})$
		$(\frac{R}{D} \times {}_{G}g_{\mu\nu})$	$(\frac{R}{D} \times {}_{\underline{G}}g_{\mu\nu})$	$(\frac{R}{D} \times g_{\mu\nu})$

3.32. Objective reality under conditions of 2 space-time dimensions

Theorem 3.49 (Objetive reality under conditions of 2 space-time dimensions)**.** *The original form of Einstein field equations (Einstein, 1916, 1917, 1950, Einstein and de Sitter, 1932) are derived as*

$$R_{\mu\nu} - \left(\left(\frac{R}{2} \right) \times g_{\mu\nu} \right) + (\Lambda \times g_{\mu\nu}) \equiv \left(\frac{8 \times \pi \times \gamma}{c^4} \right) \times T_{\mu\nu} \tag{657}$$

Under conditions of objective reality of D=2 dimensions it is

$$R_{\mu\nu} - \left(\left(\frac{R}{2} \right) \times g_{\mu\nu} \right) \equiv 0 \tag{658}$$

or

$$R_{\mu\nu} \equiv \left(\left(\frac{R}{2} \right) \times g_{\mu\nu} \right) \tag{659}$$

and the Einstein field equation's become

$$(R \times g_{\mu\nu}) - (\underline{\Lambda} \times g_{\mu\nu}) \equiv \left(\frac{4 \times \pi \times \gamma \times T}{c^4} \right) \tag{660}$$

Proof by direct proof. **If** the premise

$$\underbrace{+1 = +1}_{(Premise)} \tag{661}$$

is true, **then** the following conclusion

$$(R \times g_{\mu\nu}) - (\underline{\Lambda} \times g_{\mu\nu}) \equiv \left(\frac{4 \times \pi \times \gamma \times T}{c^4}\right) \tag{662}$$

is also true, again the absence of any technical errors presupposed. The premise

$$+1 \equiv +1 \tag{663}$$

is true. Multiplying this premise by Einstein's stress-energy tensor of general relativity, we obtain

$$+1 \times \left(\frac{8 \times \pi \times \gamma}{c^4}\right) \times \left(\frac{T}{D}\right) \times g_{\mu\nu} \equiv +1 \times \left(\frac{8 \times \pi \times \gamma}{c^4}\right) \times \left(\frac{T}{D}\right) \times g_{\mu\nu} \tag{664}$$

or

$$\left(\frac{8 \times \pi \times \gamma}{c^4}\right) \times \left(\frac{T}{D}\right) \times g_{\mu\nu} \equiv \left(\frac{8 \times \pi \times \gamma}{c^4}\right) \times \left(\frac{T}{D}\right) \times g_{\mu\nu} \tag{665}$$

According to Einstein it is $R_{\mu\nu} - \left(\left(\frac{R}{2}\right) \times g_{\mu\nu}\right) + (\Lambda \times g_{\mu\nu}) \equiv \left(\frac{8 \times \pi \times \gamma}{c^4}\right) \times T_{\mu\nu}$ (see definition 2.39). Equation 665 changes to,

$$\left(\left(\frac{R}{D}\right) \times g_{\mu\nu}\right) - \left(\left(\frac{R}{2}\right) \times g_{\mu\nu}\right) + (\Lambda \times g_{\mu\nu}) \equiv \left(\frac{8 \times \pi \times \gamma}{c^4}\right) \times \left(\frac{T}{D}\right) \times g_{\mu\nu} \tag{666}$$

which is the general form of the 2-index Einstein's field equations under conditions of D dimensions. Under conditions of D=2 space-time conditions, the 2-index Einstein's field equations becomes

$$\left(\left(\frac{R}{2}\right) \times g_{\mu\nu}\right) - \left(\left(\frac{R}{2}\right) \times g_{\mu\nu}\right) + (\Lambda \times g_{\mu\nu}) \equiv \left(\frac{8 \times \pi \times \gamma}{c^4}\right) \times \left(\frac{T}{2}\right) \times g_{\mu\nu} \tag{667}$$

or

$$0 + (\Lambda \times g_{\mu\nu}) \equiv \left(\frac{8 \times \pi \times \gamma}{c^4}\right) \times \left(\frac{T}{2}\right) \times g_{\mu\nu} \tag{668}$$

To bring it again to the point, equation 668 simplifies as

$$(\Lambda) \times g_{\mu\nu} \equiv \left(\frac{4 \times \pi \times \gamma \times T}{c^4}\right) \times g_{\mu\nu} \tag{669}$$

However, according to definition 608 it is

$$+(\Lambda \times g_{\mu\nu}) \equiv (R \times g_{\mu\nu}) - (\underline{\Lambda} \times g_{\mu\nu}) \tag{670}$$

Therefore and in general, under conditions of D=2 space-time dimensions of objective reality, the Einstein's field equations becomes

$$(R \times g_{\mu\nu}) - (\underline{\Lambda} \times g_{\mu\nu}) \equiv \left(\frac{4 \times \pi \times \gamma \times T}{c^4}\right) \tag{671}$$

In other words, our conclusion is true.

□

Remark 3.7. *We are now on the edge and only one step away from the door leading to a sudden logical collapse of Einstein's theory of general relativity. How might we save our soul from such a logical inferno? From the point of view of pure logic, the possibility of an objective reality with D=2 space-time dimensions cannot be ignored and follows straight forward in a logically consistent way from the Einstein field equations. But now, under these conditions, Einstein's cosmological constant* Λ *(see equation 669) is determined as*

$$\Lambda \equiv \left(\frac{4 \times \pi \times \gamma}{c^4}\right) \times T \tag{672}$$

or something as $\Lambda \approx T$ *and by far not a constant. Under conditions where*

$$\rho \equiv \left(\frac{T}{c^4}\right) \tag{673}$$

Einstein's cosmological constant Λ *reduces to something like a special-relativistic generalization of the Poisson's equation for gravity as*

$$\Lambda \equiv \nabla^2 \times \phi \equiv (4 \times \pi \times \gamma \times \rho) \tag{674}$$

where ∇ *is the Poisson operator and* ϕ *is a scalar potential. Whichever way one might look at this issue, it* ***cannot*** *be reasonably concluded that Einstein's cosmological constant* Λ *is a constant. In fact, if we look back to the past development of objective reality we can logically assume that before our four dimensional objective reality there could have existed an objective reality with two dimensions from which our four dimensional world has developed. However, this does not exclude, that a two-dimensional objective reality does not exist any longer. It is to be noted that a manifold with only two dimensions is determined by the stress-energy tensor of the electromagnetic field (*$b \times g_{\mu\nu}$*) given by*

$$b \times g_{\mu\nu} \equiv +\Lambda \times g_{\mu\nu} \tag{675}$$

and equally by the tensor of gravitational waves ($d \times g_{\mu\nu}$*) given by*

$$d \times g_{\mu\nu} \equiv \left(\left(\frac{R}{2}\right) \times g_{\mu\nu}\right) - \left(\Lambda \times g_{\mu\nu}\right) \tag{676}$$

		Curvature		
		YES	NO	
Momentum	YES	+0	$(+\Lambda \times g_{\mu\nu})$	$\frac{4 \times 2 \times \pi \times \gamma \times T}{2 \times c^4} \times g_{\mu\nu}$
	NO	+0	$(\frac{R}{2} \times g_{\mu\nu} - \Lambda \times g_{\mu\nu})$	$(\frac{R}{2} \times g_{\mu\nu} - \Lambda \times g_{\mu\nu})$
		+0	$\frac{R}{2} \times g_{\mu\nu}$	$\frac{R}{2} \times g_{\mu\nu}$

Table 12. Objective reality under conditions of two space-time dimensions

Theorem 3.50 (Objective reality under conditions of 1 space-time dimension)**.** *The relationship between energy, time and space is the foundation of objective reality. However, whether a one dimensional world has freed itself out of itself from the limitations of zero (dimension), the state of pure symmetry where a positive is identical with a negative or vacuum as such, might be an issue of further research. In other words, below two dimensions, it is possible that time passes over into energy and vice versa. Nonetheless, logically, the assumption of a big bang as the starting point of the development of a four-dimensional world is not completely convincing. Furthermore, whether the next step of development of objective reality is a world with 8 dimensions is a point of issue not solved today. In particular, under conditions of D=1 space-time dimension, Einstein's field equations becomes*

$$+\left(\left(\frac{R}{2}\right)\times g_{\mu\nu}\right)+\left(\Lambda\times g_{\mu\nu}\right)\equiv\left(\frac{8\times\pi\times\gamma\times T}{c^4}\right)\times g_{\mu\nu} \tag{677}$$

Proof by direct proof. **If** the premise

$$\underbrace{+1=+1}_{(Premise)} \tag{678}$$

is true, **then** the following conclusion

$$+\left(\left(\frac{R}{2}\right)\times g_{\mu\nu}\right)+\left(\Lambda\times g_{\mu\nu}\right)\equiv\left(\frac{8\times\pi\times\gamma\times T}{c^4}\right)\times g_{\mu\nu} \tag{679}$$

is also true, again the absence of any technical errors presupposed. The premise

$$+1\equiv+1 \tag{680}$$

is true. Multiplying equation 680 by the Einstein field equations, we obtain

$$\left(\left(\frac{R}{D}\right)\times g_{\mu\nu}\right)-\left(\left(\frac{R}{2}\right)\times g_{\mu\nu}\right)+\left(\Lambda\times g_{\mu\nu}\right)\equiv\left(\frac{8\times\pi\times\gamma}{c^4}\right)\times\left(\frac{T}{D}\right)\times g_{\mu\nu} \tag{681}$$

Under conditions of D=1 space-time dimension it is

$$\left(\left(\frac{R}{1}\right)\times g_{\mu\nu}\right)-\left(\left(\frac{R}{2}\right)\times g_{\mu\nu}\right)+\left(\Lambda\times g_{\mu\nu}\right)\equiv\left(\frac{8\times\pi\times\gamma}{c^4}\right)\times\left(\frac{T}{1}\right)\times g_{\mu\nu} \tag{682}$$

In general, under conditions of D=1 space-time dimension, Einstein's field equations becomes

$$+\left(\left(\frac{R}{2}\right)\times g_{\mu\nu}\right)+\left(\Lambda\times g_{\mu\nu}\right)\equiv\left(\frac{8\times\pi\times\gamma\times T}{c^4}\right)\times g_{\mu\nu} \tag{683}$$

□

Did Einstein get famous because of relativity theory, or vice versa? Did relativity theory got famous because of Einstein? On April 3, 1921, Einstein arrived in New York Harbor, USA and gave a brief explanation of his theory of relativity in an interview given to The New York Times as follows:

> "Früher hat man geglaubt, wenn alle Dinge aus der Welt verschwinden,
>
> so bleiben noch Raum und Zeit übrig.
>
> Nach der Relativitätstheorie
>
> verschwinden aber Zeit und Raum mit den Dingen."
>
> (cited according to Max Planck Institute for the History of Science (MPIWG) Einstein, 1921)

In broken English: *In former times, it was believed that when all things disappear from the world, there would still be space and time left. According to the theory of relativity, however, time and space disappear with things.*

Theorem 3.51 (Objective reality under conditions of zero space-time dimension)**.** *The relationship between energy, time and space is the foundation of objective reality.*

Proof by direct proof. **If** the premise

$$\underbrace{+1 = +1}_{(Premise)} \tag{684}$$

is true, **then** the following conclusion

$$+\left(\left(\frac{R}{2}\right) \times g_{\mu\nu}\right) + \left(\Lambda \times g_{\mu\nu}\right) \equiv \left(\frac{8 \times \pi \times \gamma \times T}{c^4}\right) \times g_{\mu\nu} \tag{685}$$

is also true, again the absence of any technical errors presupposed. The premise

$$+1 \equiv +1 \tag{686}$$

is true. Multiplying equation 686 by the Einstein field equations, we obtain

$$\left(\left(\frac{R}{D}\right) \times g_{\mu\nu}\right) - \left(\left(\frac{R}{2}\right) \times g_{\mu\nu}\right) + \left(\Lambda \times g_{\mu\nu}\right) \equiv \left(\frac{8 \times \pi \times \gamma}{c^4}\right) \times \left(\frac{T}{D}\right) \times g_{\mu\nu} \tag{687}$$

Multiplying the Einstein field equations by D, equation 687 becomes

$$\left((R) \times g_{\mu\nu}\right) - \left(\left(\frac{R \times D}{2}\right) \times g_{\mu\nu}\right) + \left(\Lambda \times D \times g_{\mu\nu}\right) \equiv \left(\frac{8 \times \pi \times \gamma}{c^4}\right) \times (T) \times g_{\mu\nu} \tag{688}$$

In point of fac, the multiplication by D is valid even under condition where D = 0 because

$$\frac{+0}{+0} \equiv +1 \tag{689}$$

((see also Barukčić, 2018, 2019b,c,d, 2020b, Barukčić and Ufuoma, 2020, Barukčić and Barukčić, 2016)). Thus far, under conditions of D=0 space-time dimension, it is

$$R \times g_{\mu\nu} - \left(\left(\frac{R \times 0}{2}\right) \times g_{\mu\nu}\right) + (\Lambda \times 0 \times g_{\mu\nu}) \equiv \left(\frac{8 \times \pi \times \gamma}{c^4}\right) \times (T) \times g_{\mu\nu} \tag{690}$$

or

$$R \times g_{\mu\nu} - \left(\left(\frac{R \times 0}{2}\right) \times g_{\mu\nu}\right) + (\Lambda \times 0 \times g_{\mu\nu}) \equiv \left(\frac{8 \times \pi \times \gamma}{c^4}\right) \times (T) \times g_{\mu\nu} \tag{691}$$

In general, under conditions of D=0 space-time dimension, the Einstein field equations become

$$R \times g_{\mu\nu} \equiv \left(\frac{8 \times \pi \times \gamma \times T}{c^4}\right) \times g_{\mu\nu} \tag{692}$$

However, according to equation 608 it is

$$(R \times g_{\mu\nu}) + 0 \equiv (\underline{\Lambda} \times g_{\mu\nu}) + (\Lambda \times g_{\mu\nu}) \tag{693}$$

Substituting this relationship into equation 692, it is

$$(R \times g_{\mu\nu}) + 0 \equiv (\underline{\Lambda} \times g_{\mu\nu}) + (\Lambda \times g_{\mu\nu}) \equiv \left(\frac{8 \times \pi \times \gamma \times T}{c^4}\right) \times g_{\mu\nu} \tag{694}$$

or

$$(\underline{\Lambda} \times g_{\mu\nu}) + (\Lambda \times g_{\mu\nu}) \equiv \left(\frac{8 \times \pi \times \gamma \times T}{c^4}\right) \times g_{\mu\nu} \tag{695}$$

There are conditions of objective reality where the stress–energy tensor vanishes, i.e. where $\left(\frac{8 \times \pi \times \gamma \times T}{c^4}\right) \times g_{\mu\nu} \equiv +0$. Under these conditions of objective reality (the stress–energy tensor vanishes), equation 695 changes to

$$(\underline{\Lambda} \times g_{\mu\nu}) + (\Lambda \times g_{\mu\nu}) \equiv \left(\frac{8 \times \pi \times \gamma \times T}{c^4}\right) \times g_{\mu\nu} = +0 \tag{696}$$

Rearranging equation 696, it is

$$+ (\Lambda \times g_{\mu\nu}) \equiv - (\underline{\Lambda} \times g_{\mu\nu}) \tag{697}$$

□

Mathematically, equation 697 is an exact solution of the Einstein field equations and describes an objective reality in which no matter or no electromagnetic fields are present, a vacuum. However, such a vacuum according to equation 697 is equally a state of pure symmetry of objective reality, where a negative is equal to a positive and vice versa. A vacuum is the unity of opposites and grounded on a contradiction.

3.35. Schrödinger Wave equation and unified field theory

Among the theoretical issues raised by the Schrödinger wave equation, despite its status as the heart of contemporary quantum mechanics, is the question, whether is it possible at all to express the Einstein field equations completely in terms of the Schrödinger equation? In other words, what objective reasons prevent us from expressing the Einstein's field equations completely in terms of a relativistic Schrödinger wave equation? Mentioned only incidentally, the definition of an expectation value of a single event forces us to consider assigning definite values to any physical quantity. Nonetheless, in what follows, we will no further touch on these topics.

Theorem 3.52 (Schrödinger Wave equation and unified field theory)**.** *For our purposes, the most important features of this theorem is the derivation of a generally covariant form of the Schrödinger equation as*

$$\left({}_RS_t \times \Psi\left({}_RS_t\right)_{\mu\nu}\right) - \left({}_Rt_t \times \Psi\left({}_Rt_t\right)_{\mu\nu}\right) \equiv \left({}_RE_t \times \Psi\left({}_RE_t\right)_{\mu\nu}\right) \tag{698}$$

Proof by direct proof. **If** the premise

$$\underbrace{+1 = +1}_{(Premise)} \tag{699}$$

is true, **then** the following conclusion

$$\left({}_{\mathrm{R}}S_{\mathrm{t}} \times \Psi\left({}_{\mathrm{R}}S_{\mathrm{t}}\right)_{\mu\nu}\right) - \left({}_{\mathrm{R}}t_{\mathrm{t}} \times \Psi\left({}_{\mathrm{R}}t_{\mathrm{t}}\right)_{\mu\nu}\right) \equiv \left({}_{\mathrm{R}}E_{\mathrm{t}} \times \Psi\left({}_{\mathrm{R}}E_{\mathrm{t}}\right)_{\mu\nu}\right) \tag{700}$$

is also true, again the absence of any technical errors presupposed. The premise

$$+1 \equiv +1 \tag{701}$$

is true. We multiply equation 701 through by equation 302. It is

$${}_{\mathrm{R}}S_{\mathrm{t}} \equiv E\left({}_{\mathrm{R}}E_{\mathrm{t}}\right) + E\left({}_{\mathrm{R}}t_{\mathrm{t}}\right) \tag{702}$$

or

$${}_{\mathrm{R}}S_{\mathrm{t}} - E\left({}_{\mathrm{R}}t_{\mathrm{t}}\right) \equiv E\left({}_{\mathrm{R}}E_{\mathrm{t}}\right) \tag{703}$$

We multiply equation 703 through by the metric tensor $g_{\mu\nu}$. Under particular conditions, the Einstein gravitational field equations become

$$\left({}_{\mathrm{R}}S_{\mathrm{t}}\right) \times g_{\mu\nu} - \left(E\left({}_{\mathrm{R}}t_{\mathrm{t}}\right)\right) \times g_{\mu\nu} \equiv \left(E\left({}_{\mathrm{R}}E_{\mathrm{t}}\right)\right) \times g_{\mu\nu} \tag{704}$$

Equation 704 changes according to equation 16 to

$$\left(\Psi\left({}_{\mathrm{R}}S_{\mathrm{t}}\right) \times {}_{\mathrm{R}}S_{\mathrm{t}} \times \Psi^{*}\left({}_{\mathrm{R}}S_{\mathrm{t}}\right)\right) \times g_{\mu\nu} - \left(\Psi\left({}_{\mathrm{R}}t_{\mathrm{t}}\right) \times {}_{\mathrm{R}}t_{\mathrm{t}} \times \Psi^{*}\left({}_{\mathrm{R}}t_{\mathrm{t}}\right)\right) \times g_{\mu\nu} \equiv \left(\Psi\left({}_{\mathrm{R}}E_{\mathrm{t}}\right) \times {}_{\mathrm{R}}E_{\mathrm{t}} \times \Psi^{*}\left({}_{\mathrm{R}}E_{\mathrm{t}}\right)\right) \times g_{\mu\nu} \tag{705}$$

Next we consider conditions where $\Psi^{*}\left({}_{\mathrm{R}}S_{\mathrm{t}}\right) \equiv \Psi^{*}\left({}_{\mathrm{R}}t_{\mathrm{t}}\right) \equiv \Psi^{*}\left({}_{\mathrm{R}}E_{\mathrm{t}}\right)$. Equation 705 changes to

$$\left(\Psi\left({}_{\mathrm{R}}S_{\mathrm{t}}\right) \times {}_{\mathrm{R}}S_{\mathrm{t}} \times \Psi^{*}\left({}_{\mathrm{R}}S_{\mathrm{t}}\right)\right) \times g_{\mu\nu} - \left(\Psi\left({}_{\mathrm{R}}t_{\mathrm{t}}\right) \times {}_{\mathrm{R}}t_{\mathrm{t}} \times \Psi^{*}\left({}_{\mathrm{R}}S_{\mathrm{t}}\right)\right) \times g_{\mu\nu} \equiv \left(\Psi\left({}_{\mathrm{R}}E_{\mathrm{t}}\right) \times {}_{\mathrm{R}}E_{\mathrm{t}} \times \Psi^{*}\left({}_{\mathrm{R}}S_{\mathrm{t}}\right)\right) \times g_{\mu\nu} \tag{706}$$

In accordance with definition 2.3 and definition 2.4, equation 706 changes to

$$
\begin{aligned}
&\left(\Psi\left({}_{\mathrm{R}}S_{\mathrm{t}}\right)_{\mu\nu} \times {}_{\mathrm{R}}S_{\mathrm{t}} \times {}^{*}\Psi\left({}_{\mathrm{R}}S_{\mathrm{t}}\right)_{\mu\nu}\right)\\
&-\left(\Psi\left({}_{\mathrm{R}}t_{\mathrm{t}}\right)_{\mu\nu} \times {}_{\mathrm{R}}t_{\mathrm{t}} \times {}^{*}\Psi\left({}_{\mathrm{R}}S_{\mathrm{t}}\right)_{\mu\nu}\right)\\
&\equiv\left(\Psi\left({}_{\mathrm{R}}E_{\mathrm{t}}\right)_{\mu\nu} \times {}_{\mathrm{R}}E_{\mathrm{t}} \times {}^{*}\Psi\left({}_{\mathrm{R}}S_{\mathrm{t}}\right)_{\mu\nu}\right)
\end{aligned}
\tag{707}
$$

We carry out the counter operation of the term ${}^{*}\Psi\left({}_{\mathrm{R}}S_{\mathrm{t}}\right)_{\mu\nu}$. After appropriate calculations, equation 707 changes to a Schrödinger wave equation of the gravitational field as

$$
\left(\Psi\left({}_{\mathrm{R}}S_{\mathrm{t}}\right)_{\mu\nu} \times {}_{\mathrm{R}}S_{\mathrm{t}}\right)-\left(\Psi\left({}_{\mathrm{R}}t_{\mathrm{t}}\right)_{\mu\nu} \times {}_{\mathrm{R}}t_{\mathrm{t}}\right) \equiv \left(\Psi\left({}_{\mathrm{R}}E_{\mathrm{t}}\right)_{\mu\nu} \times {}_{\mathrm{R}}E_{\mathrm{t}}\right) \tag{708}
$$

□

Nonetheless, experience also teaches us that the shorter the line of arguments, the better. The foundation of the Einstein field equations has been identified (see equation 573) as

$$
\frac{R}{D}-\left(\frac{R}{2}\right)+(\Lambda) \equiv \left(\frac{8 \times \pi \times \gamma \times T}{c^{4} \times D}\right) \tag{709}
$$

Multiplying equation 709 by wave function tensor (see definition 709), it is

$$
\left(\frac{R}{D} \times \Psi\left({}_{\mathrm{R}}S_{\mathrm{t}}\right)_{\mu\nu}\right)-\left(\left(\left(\frac{R}{2}\right)-(\Lambda)\right) \times \Psi\left({}_{\mathrm{R}}S_{\mathrm{t}}\right)_{\mu\nu}\right) \equiv \left(\left(\frac{8 \times \pi \times \gamma \times T}{c^{4} \times D}\right) \times \Psi\left({}_{\mathrm{R}}S_{\mathrm{t}}\right)_{\mu\nu}\right) \tag{710}
$$

Under conditions of normalization, examined even in tough conditions, it is that

$$
\Psi\left({}_{\mathrm{R}}S_{\mathrm{t}}\right)_{\mu\nu} \cap {}^{*}\Psi\left({}_{\mathrm{R}}S_{\mathrm{t}}\right)_{\mu\nu} \equiv 1_{\mu\nu} \tag{711}
$$

where $\cap$ denotes the commutative multiplication of tensors. From equation 711 follows that

$$
{}^{*}\Psi\left({}_{\mathrm{R}}S_{\mathrm{t}}\right)_{\mu\nu} \equiv \frac{1_{\mu\nu}}{\Psi\left({}_{\mathrm{R}}S_{\mathrm{t}}\right)_{\mu\nu}} \tag{712}
$$

Theorem 3.53 (CAUSAL RELATIONSHIP K UNDER CONDITIONS OF THE THEORY OF GENERAL RELATIVITY).

The world we are living in sees itself as charged with remaking itself, and doubtless feels the need to change itself permanently. However, these changes don't have to happen for the better. Does objective reality posses any mechanism in preventing itself from destroying itself? What is keeping this world from destroying itself?

In this context, let $p({}_{R}U_{kl\mu\nu\ldots})$ represent the probability tensor of the tensor of cause ${}_{R}U_{kl\mu\nu\ldots}$. Let ${}^{2}E({}_{R}U_{kl\mu\nu\ldots})$ denote the expectation value of the cause ${}^{2}{}_{R}U_{kl\mu\nu\ldots}$. Let $E({}_{R}U_{kl\mu\nu\ldots})$ denote the expectation value of the cause ${}_{R}U_{kl\mu\nu\ldots}$. Let $\sigma({}_{R}U_{kl\mu\nu\ldots})$ denote the standard deviation of the cause ${}_{R}U_{kl\mu\nu\ldots}$. Let ${}^{2}\sigma({}_{R}U_{kl\mu\nu\ldots})$ denote the variance of the cause ${}_{R}U_{kl\mu\nu\ldots}$.

Let $p({}_{R}W_{kl\mu\nu\ldots})$ represent the probability tensor of the tensor of its own effect ${}_{R}W_{kl\mu\nu\ldots}$. Let ${}^{2}E({}_{R}W_{kl\mu\nu\ldots})$ denote the expectation value of the effect ${}^{2}{}_{R}W_{kl\mu\nu\ldots}$. Let $E({}_{R}W_{kl\mu\nu\ldots})$ denote the expectation value of the effect ${}_{R}W_{kl\mu\nu\ldots}$. Let $\sigma({}_{R}W_{kl\mu\nu\ldots})$ denote the standard deviation of the effect ${}_{R}W_{kl\mu\nu\ldots}$. Let ${}^{2}\sigma({}_{R}W_{kl\mu\nu\ldots})$ denote the variance of the effect ${}_{R}W_{kl\mu\nu\ldots}$.

Let $\sigma({}_{R}U_{kl\mu\nu\ldots}, {}_{R}W_{kl\mu\nu\ldots})$ denote the co-variance of cause ${}_{R}U_{kl\mu\nu\ldots}$ and effect ${}_{R}W_{kl\mu\nu\ldots}$.

$$\begin{aligned}
&k\left({}_{R}U_{kl\mu\nu\ldots}, {}_{R}W_{kl\mu\nu\ldots}\right)\\
&\equiv \frac{\sigma\left({}_{R}U_{kl\mu\nu\ldots}, {}_{R}W_{kl\mu\nu\ldots}\right)}{\sigma({}_{R}U_{kl\mu\nu\ldots}) \cap \sigma({}_{R}W_{kl\mu\nu\ldots})}\\
&\equiv \frac{\left(p\left({}_{R}U_{kl\mu\nu\ldots}, {}_{R}W_{kl\mu\nu\ldots}\right) - \left(p\left({}_{R}U_{kl\mu\nu\ldots}\right) \cap p\left({}_{R}W_{kl\mu\nu\ldots}\right)\right)\right)}{\sqrt[2]{p\left({}_{R}U_{kl\mu\nu\ldots}\right) \cap \left(1_{kl\mu\nu\ldots} - p\left({}_{R}U_{kl\mu\nu\ldots}\right)\right) \cap p\left({}_{R}W_{kl\mu\nu\ldots}\right) \cap \left(1_{kl\mu\nu\ldots} - p\left({}_{R}W_{kl\mu\nu\ldots}\right)\right)}}
\end{aligned} \tag{713}$$

by modus ponens. **If** the premise

$$\underbrace{+1 = +1}_{(Premise)} \tag{714}$$

is true, **then** the conclusion

$$k\left({}_{R}U_{kl\mu\nu\ldots}, {}_{R}W_{kl\mu\nu\ldots}\right) \equiv \frac{\sigma\left({}_{R}U_{kl\mu\nu\ldots}, {}_{R}W_{kl\mu\nu\ldots}\right)}{\sigma({}_{R}U_{kl\mu\nu\ldots}) \cap \sigma({}_{R}W_{kl\mu\nu\ldots})} \tag{715}$$

is also true, the absence of any technical errors presupposed. The premise

$$+1 \equiv +1 \tag{716}$$

is true. Multiplying this premise (i.e. axiom) by $\left({}_{R}U_{kl\mu\nu\ldots} \cap {}_{R}W_{kl\mu\nu\ldots}\right)$ it is

$$\left({}_{\mathrm{R}}U_{\mathrm{kl}\mu\nu\ldots} \cap {}_{\mathrm{R}}W_{\mathrm{kl}\mu\nu\ldots}\right) \equiv \left({}_{\mathrm{R}}U_{\mathrm{kl}\mu\nu\ldots} \cap {}_{\mathrm{R}}W_{\mathrm{kl}\mu\nu\ldots}\right) \tag{717}$$

According to equation 109 it is

$${}_{\mathrm{R}}U_{\mathrm{kl}\mu\nu\ldots} \equiv \frac{\sigma\left({}_{\mathrm{R}}U_{\mathrm{kl}\mu\nu\ldots}\right)}{\sqrt[2]{p\left({}_{\mathrm{R}}U_{\mathrm{kl}\mu\nu\ldots}\right) \cap \left(1_{\mathrm{kl}\mu\nu\ldots} - p\left({}_{\mathrm{R}}U_{\mathrm{kl}\mu\nu\ldots}\right)\right)}} \tag{718}$$

Equation 717 changes to

$$\begin{aligned} &\left({}_{\mathrm{R}}U_{\mathrm{kl}\mu\nu\ldots} \cap {}_{\mathrm{R}}W_{\mathrm{kl}\mu\nu\ldots}\right) \\ &\equiv \left(\frac{\sigma\left({}_{\mathrm{R}}U_{\mathrm{kl}\mu\nu\ldots}\right)}{\sqrt[2]{p\left({}_{\mathrm{R}}U_{\mathrm{kl}\mu\nu\ldots}\right) \cap \left(1_{\mathrm{kl}\mu\nu\ldots} - p\left({}_{\mathrm{R}}U_{\mathrm{kl}\mu\nu\ldots}\right)\right)}}\right) \\ &\cap {}_{\mathrm{R}}W_{\mathrm{kl}\mu\nu\ldots} \end{aligned} \tag{719}$$

Additionally, according to equation 109 it is

$${}_{\mathrm{R}}W_{\mathrm{kl}\mu\nu\ldots} \equiv \frac{\sigma\left({}_{\mathrm{R}}W_{\mathrm{kl}\mu\nu\ldots}\right)}{\sqrt[2]{p\left({}_{\mathrm{R}}W_{\mathrm{kl}\mu\nu\ldots}\right) \cap \left(1_{\mathrm{kl}\mu\nu\ldots} - p\left({}_{\mathrm{R}}W_{\mathrm{kl}\mu\nu\ldots}\right)\right)}} \tag{720}$$

Equation 719 changes to

$$\begin{aligned} &\left({}_{\mathrm{R}}U_{\mathrm{kl}\mu\nu\ldots} \cap {}_{\mathrm{R}}W_{\mathrm{kl}\mu\nu\ldots}\right) \\ &\equiv \left(\frac{\sigma\left({}_{\mathrm{R}}U_{\mathrm{kl}\mu\nu\ldots}\right)}{\sqrt[2]{p\left({}_{\mathrm{R}}U_{\mathrm{kl}\mu\nu\ldots}\right) \cap \left(1_{\mathrm{kl}\mu\nu\ldots} - p\left({}_{\mathrm{R}}U_{\mathrm{kl}\mu\nu\ldots}\right)\right)}}\right) \\ &\cap \left(\frac{\sigma\left({}_{\mathrm{R}}W_{\mathrm{kl}\mu\nu\ldots}\right)}{\sqrt[2]{p\left({}_{\mathrm{R}}W_{\mathrm{kl}\mu\nu\ldots}\right) \cap \left(1_{\mathrm{kl}\mu\nu\ldots} - p\left({}_{\mathrm{R}}W_{\mathrm{kl}\mu\nu\ldots}\right)\right)}}\right) \end{aligned} \tag{721}$$

According to definition 2.35, equation 116, it is

$$\begin{aligned} &{}_{\mathrm{R}}U_{\mathrm{kl}\mu\nu\ldots} \cap {}_{\mathrm{R}}W_{\mathrm{kl}\mu\nu\ldots} \\ &\equiv \frac{\sigma\left({}_{\mathrm{R}}U_{\mathrm{kl}\mu\nu\ldots}, {}_{\mathrm{R}}W_{\mathrm{kl}\mu\nu\ldots}\right)}{\left(p\left({}_{\mathrm{R}}U_{\mathrm{kl}\mu\nu\ldots}, {}_{\mathrm{R}}W_{\mathrm{kl}\mu\nu\ldots}\right) - \left(p\left({}_{\mathrm{R}}U_{\mathrm{kl}\mu\nu\ldots}\right) \cap p\left({}_{\mathrm{R}}W_{\mathrm{kl}\mu\nu\ldots}\right)\right)\right)} \end{aligned} \tag{722}$$

Simplifying equation 721, we obtain

$$\begin{aligned} &\left(\frac{\sigma\left({}_{\mathrm{R}}U_{\mathrm{kl}\mu\nu\ldots}, {}_{\mathrm{R}}W_{\mathrm{kl}\mu\nu\ldots}\right)}{\left(p\left({}_{\mathrm{R}}U_{\mathrm{kl}\mu\nu\ldots}, {}_{\mathrm{R}}W_{\mathrm{kl}\mu\nu\ldots}\right) - \left(p\left({}_{\mathrm{R}}U_{\mathrm{kl}\mu\nu\ldots}\right) \cap p\left({}_{\mathrm{R}}W_{\mathrm{kl}\mu\nu\ldots}\right)\right)\right)}\right) \\ &\equiv \left(\frac{\sigma\left({}_{\mathrm{R}}U_{\mathrm{kl}\mu\nu\ldots}\right)}{\sqrt[2]{p\left({}_{\mathrm{R}}U_{\mathrm{kl}\mu\nu\ldots}\right) \cap \left(1_{\mathrm{kl}\mu\nu\ldots} - p\left({}_{\mathrm{R}}U_{\mathrm{kl}\mu\nu\ldots}\right)\right)}}\right) \\ &\cap \left(\frac{\sigma\left({}_{\mathrm{R}}W_{\mathrm{kl}\mu\nu\ldots}\right)}{\sqrt[2]{p\left({}_{\mathrm{R}}W_{\mathrm{kl}\mu\nu\ldots}\right) \cap \left(1 - p\left({}_{\mathrm{R}}W_{\mathrm{kl}\mu\nu\ldots}\right)\right)}}\right) \end{aligned} \tag{723}$$

Further rearrangement of equation 723 yields the causal relationship between the cause ${}_{\mathrm{R}}\mathrm{U}_{\mathrm{kl}\mu\nu\ldots}$ and the effect ${}_{\mathrm{R}}\mathrm{W}_{\mathrm{kl}\mu\nu\ldots}$, denoted as $k\left({}_{\mathrm{R}}U_{\mathrm{kl}\mu\nu\ldots}, {}_{\mathrm{R}}W_{\mathrm{kl}\mu\nu\ldots}\right)$, as

$$
\begin{aligned}
& k\left({}_{\mathrm{R}}U_{\mathrm{kl}\mu\nu\ldots}, {}_{\mathrm{R}}W_{\mathrm{kl}\mu\nu\ldots}\right) \\
& \equiv \frac{\sigma\left({}_{\mathrm{R}}U_{\mathrm{kl}\mu\nu\ldots}, {}_{\mathrm{R}}W_{\mathrm{kl}\mu\nu\ldots}\right)}{\sigma({}_{\mathrm{R}}U_{\mathrm{kl}\mu\nu\ldots}) \cap \sigma({}_{\mathrm{R}}W_{\mathrm{kl}\mu\nu\ldots})} \\
& \equiv \frac{\left(p\left({}_{\mathrm{R}}U_{\mathrm{kl}\mu\nu\ldots}, {}_{\mathrm{R}}W_{\mathrm{kl}\mu\nu\ldots}\right) - \left(p\left({}_{\mathrm{R}}U_{\mathrm{kl}\mu\nu\ldots}\right) \cap p\left({}_{\mathrm{R}}W_{\mathrm{kl}\mu\nu\ldots}\right)\right)\right)}{\sqrt[2]{p\left({}_{\mathrm{R}}U_{\mathrm{kl}\mu\nu\ldots}\right) \cap \left(1_{\mathrm{kl}\mu\nu\ldots} - p\left({}_{\mathrm{R}}U_{\mathrm{kl}\mu\nu\ldots}\right)\right) \cap p\left({}_{\mathrm{R}}W_{\mathrm{kl}\mu\nu\ldots}\right) \cap \left(1_{\mathrm{kl}\mu\nu\ldots} - p\left({}_{\mathrm{R}}W_{\mathrm{kl}\mu\nu\ldots}\right)\right)}}
\end{aligned}
\tag{724}
$$

□

Under conditions of Einstein's general theory of relativity, the causal relationship k (**Einstein's Weltformel**), denoted as $k\left({}_{\mathrm{R}}U_{\mathrm{kl}\mu\nu\ldots}, {}_{\mathrm{R}}W_{\mathrm{kl}\mu\nu\ldots}\right)$, is given by

$$
\begin{aligned}
& k\left({}_{\mathrm{R}}U_{\mathrm{kl}\mu\nu\ldots}, {}_{\mathrm{R}}W_{\mathrm{kl}\mu\nu\ldots}\right) \\
& \equiv \frac{\sigma\left({}_{\mathrm{R}}U_{\mathrm{kl}\mu\nu\ldots}, {}_{\mathrm{R}}W_{\mathrm{kl}\mu\nu\ldots}\right)}{\sqrt[2]{{}^{2}\sigma\left({}_{\mathrm{R}}U_{\mathrm{kl}\mu\nu\ldots}\right) \cap {}^{2}\sigma\left({}_{\mathrm{R}}W_{\mathrm{kl}\mu\nu\ldots}\right)}} \\
& \equiv \frac{\sigma\left({}_{\mathrm{R}}U_{\mathrm{kl}\mu\nu\ldots}, {}_{\mathrm{R}}W_{\mathrm{kl}\mu\nu\ldots}\right)}{\sigma\left({}_{\mathrm{R}}U_{\mathrm{kl}\mu\nu\ldots}\right) \cap \sigma\left({}_{\mathrm{R}}W_{\mathrm{kl}\mu\nu\ldots}\right)} \\
& \equiv \frac{\left({}_{\mathrm{R}}U_{\mathrm{kl}\mu\nu\ldots} \cap {}_{\mathrm{R}}W_{\mathrm{kl}\mu\nu\ldots}\right) \cap \left(p\left({}_{\mathrm{R}}U_{\mathrm{kl}\mu\nu\ldots}, {}_{\mathrm{R}}W_{\mathrm{kl}\mu\nu\ldots}\right) - \left(p\left({}_{\mathrm{R}}U_{\mathrm{kl}\mu\nu\ldots}\right) \cap p\left({}_{\mathrm{R}}W_{\mathrm{kl}\mu\nu\ldots}\right)\right)\right)}{\sqrt[2]{\left(\left({}_{\mathrm{R}}U_{\mathrm{kl}\mu\nu\ldots}{}^{2}\right) \cap \left(p\left({}_{\mathrm{R}}U_{\mathrm{kl}\mu\nu\ldots}\right) \cap \left(1_{\mathrm{kl}\mu\nu\ldots} - p\left({}_{\mathrm{R}}U_{\mathrm{kl}\mu\nu\ldots}\right)\right)\right) \cap \left({}_{\mathrm{R}}W_{\mathrm{kl}\mu\nu\ldots}{}^{2}\right) \cap \left(p\left({}_{\mathrm{R}}W_{\mathrm{kl}\mu\nu\ldots}\right) \cap \left(1_{\mathrm{kl}\mu\nu\ldots} - p\left({}_{\mathrm{R}}W_{\mathrm{kl}\mu\nu\ldots}\right)\right)\right)\right)}} \\
& \equiv \frac{\left({}_{\mathrm{R}}U_{\mathrm{kl}\mu\nu\ldots} \cap {}_{\mathrm{R}}W_{\mathrm{kl}\mu\nu\ldots}\right) \cap \left(p\left({}_{\mathrm{R}}U_{\mathrm{kl}\mu\nu\ldots}, {}_{\mathrm{R}}W_{\mathrm{kl}\mu\nu\ldots}\right) - \left(p\left({}_{\mathrm{R}}U_{\mathrm{kl}\mu\nu\ldots}\right) \cap p\left({}_{\mathrm{R}}W_{\mathrm{kl}\mu\nu\ldots}\right)\right)\right)}{\left({}_{\mathrm{R}}U_{\mathrm{kl}\mu\nu\ldots} \cap {}_{\mathrm{R}}W_{\mathrm{kl}\mu\nu\ldots}\right) \cap \sqrt[2]{\left(\left(p\left({}_{\mathrm{R}}U_{\mathrm{kl}\mu\nu\ldots}\right) \cap \left(1_{\mathrm{kl}\mu\nu\ldots} - p\left({}_{\mathrm{R}}U_{\mathrm{kl}\mu\nu\ldots}\right)\right)\right) \cap \left(p\left({}_{\mathrm{R}}W_{\mathrm{kl}\mu\nu\ldots}\right) \cap \left(1_{\mathrm{kl}\mu\nu\ldots} - p\left({}_{\mathrm{R}}W_{\mathrm{kl}\mu\nu\ldots}\right)\right)\right)\right)}} \\
& \equiv \frac{\left(p\left({}_{\mathrm{R}}U_{\mathrm{kl}\mu\nu\ldots}, {}_{\mathrm{R}}W_{\mathrm{kl}\mu\nu\ldots}\right) - \left(p\left({}_{\mathrm{R}}U_{\mathrm{kl}\mu\nu\ldots}\right) \cap p\left({}_{\mathrm{R}}W_{\mathrm{kl}\mu\nu\ldots}\right)\right)\right)}{\sqrt[2]{\left(\left(p\left({}_{\mathrm{R}}U_{\mathrm{kl}\mu\nu\ldots}\right) \cap \left(1_{\mathrm{kl}\mu\nu\ldots} - p\left({}_{\mathrm{R}}U_{\mathrm{kl}\mu\nu\ldots}\right)\right)\right) \cap \left(p\left({}_{\mathrm{R}}W_{\mathrm{kl}\mu\nu\ldots}\right) \cap \left(1_{\mathrm{kl}\mu\nu\ldots} - p\left({}_{\mathrm{R}}W_{\mathrm{kl}\mu\nu\ldots}\right)\right)\right)\right)}}
\end{aligned}
\tag{725}
$$

♡

3.37. Relativistic Doppler effect

A frame of reference which is moving with an (internal, emitter, sender) observer at rest relative to an event, a quantum mechanical entity et cetera is denoted by the sign 0. A frame of reference which is moving and at rest with an (external) observer relative to an event, a quantum mechanical entity et cetera is denoted by the sign R (stationary of relativistic observer, receiver). In general, different observers which are stationary relative to each other are sharing the same frame of reference. However, the reference frames are no longer equivalent under conditions where one observer is moving uniformly or even accelerating relative to another. A parameter which is invariant will take the same value or form in every inertial frame reference. In this context, let ${}_{0}c_{t}$ denote the speed of the light in vacuum as determined by the co-moving observer, let ${}_{0}f_{t}$ denote the frequency of the light in vacuum as determined by the co-moving observer (i.e. the emitted frequency), let ${}_{0}\lambda_{t}$ denote the wave-length of the light in vacuum as determined by the co-moving observer (i. e. the emitted wave-length). In general, it is

$${}_{0}c_{t} \equiv {}_{0}f_{t} \times {}_{0}\lambda_{t} \tag{726}$$

Furthermore, let ${}_{R}c_{t}$ denote the speed of the light in vacuum as determined by the (external) stationary observer, let ${}_{R}f_{t}$ denote the frequency of the light in vacuum as determined by the (external) stationary observer (i.e. the observed frequency, the frequency measured which comes from a source 0), let ${}_{R}\lambda_{t}$ denote the wave-length of the light in vacuum as determined by the (external) stationary observer (i.e. the observed wave-length). In general, it is

$${}_{R}c_{t} \equiv {}_{R}f_{t} \times {}_{R}\lambda_{t} \tag{727}$$

Following Einstein, there are conditions where both observers, the co-moving and the stationary observer will agree on the value of the speed of the light in vacuum. In other words, there are conditions(Einstein, 1905d) where

$${}_{0}c_{t} \equiv {}_{R}c_{t} \tag{728}$$

It might remain an open issue whether equation 728 is valid in general(Einstein, 1911, 1912a) and without any exemption. However, and in contrast to equation 728, both observer need not agree, neither on the frequency nor on the wave-length. In physics and astronomy, a change of the wavelength (an increase (i.e. redshift) or a decrease (blueshift)) is often denoted by the letter z.

Theorem 3.54 (The value of z)**.** *In general, it is*

$$+1 \equiv +1 \tag{729}$$

Proof by direct proof. The premise

$$+1 \equiv +1 \tag{730}$$

is true. In the following, we rearrange the premise. We obtain

$${}_{R}\lambda_{t} \equiv {}_{R}\lambda_{t} \tag{731}$$

Adding 0, equation 731 becomes,

$$_{R}\lambda_{t} - {_{0}\lambda_{t}} + {_{0}\lambda_{t}} \equiv {_{R}\lambda_{t}} \tag{732}$$

In our understanding, it is $_{0}\underline{\lambda}_{t} \equiv {_{R}\lambda_{t}} - {_{0}\lambda_{t}}$. Equation 732 becomes

$$_{0}\underline{\lambda}_{t} + {_{0}\lambda_{t}} \equiv {_{R}\lambda_{t}} \tag{733}$$

Dividing equation 733 by the term $_{0}\lambda_{t}$, it is

$$\frac{_{0}\underline{\lambda}_{t}}{_{0}\lambda_{t}} + \frac{_{0}\lambda_{t}}{_{0}\lambda_{t}} \equiv \frac{_{R}\lambda_{t}}{_{0}\lambda_{t}} \tag{734}$$

Equation 734 simplifies as

$$\frac{_{0}\underline{\lambda}_{t}}{_{0}\lambda_{t}} + 1 \equiv \frac{_{R}\lambda_{t}}{_{0}\lambda_{t}} \tag{735}$$

In general, we obtain the z value as

$$z \equiv \frac{_{0}\underline{\lambda}_{t}}{_{0}\lambda_{t}} \equiv \frac{_{R}\lambda_{t}}{_{0}\lambda_{t}} - 1 \tag{736}$$

□

A source 0 emitting a wavelength which is moving away from an observer R (receiver), leads to a redshift (z > 0). A source 0 emitting a wavelength which moves towards the observer R (receiver), leads to a blueshift (z < 0). Meanwhile, on January 27, 2011, Bouwens et al. (see Bouwens et al., 2011) identified UDFj-39546284, an object located in the Fornax constellation with a z-value of about z=10.3 and potentially z = 11.9. The light needed more than 13.2 billion years to reach the Hubble Space Telescope in earth's orbit.

Theorem 3.55. *The Austrian mathematician and physicist Christian Andreas Doppler (1803-1853) discovered the Doppler effect(see Doppler, 1842) in the year 1842. Soon, the Dutch Christophorus Henricus Diedericus Buys Ballot (1817–1890) confirmed(see Ballot, 1845) Doppler's effect experimentally in the year 1845. In general, Doppler's effect follows from special theory of relativity, as*

$$\frac{_{0}f_{t}{}^{2}}{_{R}f_{t}{}^{2}} \equiv \left(1 - \frac{v^{2}}{c^{2}}\right) \tag{737}$$

Proof by direct proof. The premise

$$+1 \equiv +1 \tag{738}$$

is true. In the following, we rearrange the premise. We obtain (see equation 240 and equation 241)

$$\frac{_{0}E_{t}{}^{2}}{_{R}E_{t}{}^{2}} \equiv \left(1 - \frac{v^{2}}{c^{2}}\right) \tag{739}$$

According to the Planck-Einstein energy–frequency relation(see Einstein, 1905a, Planck, 1901), it is

$$_{0}E_{t}{}^{2} = {_{0}h_{t}}^{2} \times {_{0}f_{t}}^{2} \tag{740}$$

where ${}_{0}\mathrm{E}_{\mathrm{t}}$ is the energy of a quantum mechanical entity as determined by the co-moving observer (i.e. the source which is emitting the wave), ${}_{0}\mathrm{h}_{\mathrm{t}}$ is Planck's constant h as determined by the co-moving observer (i.e. the source emitting the wave) and equally

$$ {}_{\mathrm{R}}E_{\mathrm{t}}{}^{2} = {}_{\mathrm{R}}h_{\mathrm{t}}{}^{2} \times {}_{\mathrm{R}}f_{\mathrm{t}}{}^{2} \tag{741} $$

where ${}_{\mathrm{R}}\mathrm{E}_{\mathrm{t}}$ is the energy of a quantum mechanical entity as determined by the stationary observer (i.e. the observer which is receiving the wave), ${}_{\mathrm{R}}\mathrm{h}_{\mathrm{t}}$ is Planck's constant h as determined by the stationary observer (i.e. the observer receiving the wave). Equation 739 (see equation 740 and equation 741) simplifies as

$$ \frac{{}_{0}h_{\mathrm{t}}{}^{2} \times {}_{0}f_{\mathrm{t}}{}^{2}}{{}_{\mathrm{R}}h_{\mathrm{t}}{}^{2} \times {}_{\mathrm{R}}f_{\mathrm{t}}{}^{2}} \equiv \left(1 - \frac{v^2}{c^2}\right) \tag{742} $$

Under conditions where ${}_{0}h_{\mathrm{t}}{}^{2} \equiv {}_{\mathrm{R}}h_{\mathrm{t}}{}^{2}$, equation 742 becomes (according to equation 726 and equation 727 and equation 736)

$$ \frac{{}_{0}f_{\mathrm{t}}{}^{2}}{{}_{\mathrm{R}}f_{\mathrm{t}}{}^{2}} \equiv \frac{{}_{\mathrm{R}}\lambda_{\mathrm{t}}{}^{2}}{{}_{0}\lambda_{\mathrm{t}}{}^{2}} \equiv \left(1 - \frac{v^2}{c^2}\right) \tag{743} $$

□

However, determining the relativistic (transverse) Doppler effect(Einstein, 1907) derived as

$$ \frac{{}_{0}f_{\mathrm{t}}{}^{2}}{{}_{\mathrm{R}}f_{\mathrm{t}}{}^{2}} \equiv \frac{{}_{\mathrm{R}}\lambda_{\mathrm{t}}{}^{2}}{{}_{0}\lambda_{\mathrm{t}}{}^{2}} \equiv \left(1 - \frac{v^2}{c^2}\right) \tag{744} $$

by experiments need not be identical with determining the classical Doppler effect. The relative velocity of a distant object (with respect to our earth) can be calculated in accordance with the Doppler effect by measuring the spectral lines in the spectrum of the (distant) object.

*3.38. **Without** momentum **no** curvature*

Theorem 3.56 (**Without** momentum **no** curvature). *Momentum is determined by different factors. However, momentum itself is a determining part of changes too. There are conditions in nature where without momentum, no curvature will be given. Without exception, these conditions in nature demand that*

$$ b_{\mu\nu} \equiv \Lambda \times g_{\mu\nu} \tag{745} $$

Proof by direct proof. The premise

$$ +1 \equiv +1 \tag{746} $$

is true. In the following, we rearrange the premise. We obtain

$$ c_{\mu\nu} \equiv (b_{\mu\nu} - \Lambda \times g_{\mu\nu}) \equiv 0 \tag{747} $$

From equation 747 follows that

$$b_{\mu\nu} \equiv \Lambda \times g_{\mu\nu} \tag{748}$$

□

Table 13 is illustrating equation 748 in more detail.

		Curvature		
		YES	NO	
Momentum	YES	$\left\langle G_{\mu\nu} \right\rangle$	$\left\langle \Lambda \times g_{\mu\nu} \right\rangle$	$\left\langle \frac{8 \times \pi \times \gamma}{c^4 \times D} \times g_{\mu\nu} \right\rangle$
	NO	$\left\langle 0 \right\rangle$	$\left\langle (\frac{R}{2} \times g_{\mu\nu} - (\Lambda \times g_{\mu\nu})) \right\rangle$	$\left\langle \left(\frac{R}{2} - \Lambda\right) \times g_{\mu\nu} \right\rangle$
		$\left\langle G_{\mu\nu} \equiv \left(\frac{R}{D} - \frac{R}{2}\right) \times g_{\mu\nu} \right\rangle$	$\left\langle \frac{R}{2} \times g_{\mu\nu} \right\rangle$	$\left\langle R_{\mu\nu} \equiv \frac{R}{D} \times g_{\mu\nu} \right\rangle$

Table 13. Without momentum, **no** curvature.

3.39. *If momentum then curvature*

Theorem 3.57 (**If** momentum **then** curvature)**.** *There are circumstances in nature where momentum implies curvature. These conditions demand that*

$$c_{\mu\nu} \equiv -\Lambda \times g_{\mu\nu} \tag{749}$$

Proof by direct proof. The premise

$$+1 \equiv +1 \tag{750}$$

is true. In the following, we rearrange the premise. We obtain

$$b_{\mu\nu} \equiv (c_{\mu\nu} + \Lambda \times g_{\mu\nu}) \equiv 0 \tag{751}$$

From equation 751 follows that

$$c_{\mu\nu} \equiv -\Lambda \times g_{\mu\nu} \tag{752}$$

□

Table 14 is illustrating equation 752 in more detail.

		Curvature YES	Curvature NO	
Momentum	YES	$\left\langle \frac{8 \times \pi \times \gamma}{c^4 \times D} \times g_{\mu\nu} \right\rangle$	$\left\langle 0 \right\rangle$	$\left\langle \frac{8 \times \pi \times \gamma}{c^4 \times D} \times g_{\mu\nu} \right\rangle$
	NO	$\left\langle -\Lambda \times g_{\mu\nu} \right\rangle$	$\left\langle \frac{R}{2} \times g_{\mu\nu} \right\rangle$	$\left\langle \left(\frac{R}{2} - \Lambda \right) \times g_{\mu\nu} \right\rangle$
		$\left\langle G_{\mu\nu} \equiv \left(\frac{R}{D} - \frac{R}{2} \right) \times g_{\mu\nu} \right\rangle$	$\left\langle \frac{R}{2} \times g_{\mu\nu} \right\rangle$	$\left\langle R_{\mu\nu} \equiv \frac{R}{D} \times g_{\mu\nu} \right\rangle$

Table 14. If momentum **then** curvature

3.40. Momentum ***excludes*** *curvature* ***and vice versa***

Theorem 3.58 (Momentum **excludes** curvature **and vice versa**)**.** *Conditions in nature where momentum excludes curvature and vice versa demand that*

$$c_{\mu\nu} \equiv G_{\mu\nu} \tag{753}$$

and that

$$b_{\mu\nu} \equiv \frac{8 \times \pi \times \gamma}{c^4 \times D} \times g_{\mu\nu} \tag{754}$$

while

$$a_{\mu\nu} \equiv 0 \tag{755}$$

Proof by direct proof. The premise

$$+1 \equiv +1 \tag{756}$$

is true. In the following, we rearrange the premise. We obtain

$$a_{\mu\nu} \equiv (G_{\mu\nu} - c_{\mu\nu}) \equiv \left(\frac{8 \times \pi \times \gamma}{c^4 \times D} \times g_{\mu\nu} \right) - b_{\mu\nu} \equiv 0 \tag{757}$$

From equation 757 follows that

$$G_{\mu\nu} \equiv c_{\mu\nu} \tag{758}$$

and that

$$b_{\mu\nu} \equiv \left(\frac{8 \times \pi \times \gamma}{c^4 \times D} \times g_{\mu\nu} \right) \tag{759}$$

□

Table 15 is illustrating equation 758 and equation 759 in more detail.

		Curvature YES	Curvature NO	
Momentum	YES	$\left\langle 0 \right\rangle$	$\left\langle \frac{8 \times \pi \times \gamma}{c^4 \times D} \times g_{\mu\nu} \right\rangle$	$\left\langle \frac{8 \times \pi \times \gamma}{c^4 \times D} \times g_{\mu\nu} \right\rangle$
	NO	$\left\langle \frac{8 \times \pi \times \gamma}{c^4 \times D} \times g_{\mu\nu} - \Lambda \times g_{\mu\nu} \right\rangle$	$\left\langle \frac{R}{2} \times g_{\mu\nu} - \frac{8 \times \pi \times \gamma}{c^4 \times D} \times g_{\mu\nu} \right\rangle$	$\left\langle \left(\frac{R}{2} - \Lambda\right) \times g_{\mu\nu} \right\rangle$
		$\left\langle G_{\mu\nu} \equiv \left(\frac{R}{D} - \frac{R}{2}\right) \times g_{\mu\nu} \right\rangle$	$\left\langle \frac{R}{2} \times g_{\mu\nu} \right\rangle$	$\left\langle R_{\mu\nu} \equiv \frac{R}{D} \times g_{\mu\nu} \right\rangle$

Table 15. Momentum **excludes** curvature **and vice versa**

3.41. ***Either*** *momentum* ***or*** *curvature*

Either momentum or curvature conditions(Barukčić, 2016b) demand that

		Curvature YES	Curvature NO	
Momentum	YES	$\left\langle 0 \right\rangle$	$\left\langle \left(\frac{R}{2}\right) \times g_{\mu\nu} \right\rangle$	$\left\langle \frac{8 \times \pi \times \gamma}{c^4 \times D} \times g_{\mu\nu} \right\rangle$
	NO	$\left\langle \left(\frac{R}{2} - \Lambda\right) \times g_{\mu\nu} \right\rangle$	$\left\langle 0 \right\rangle$	$\left\langle \left(\frac{R}{2} - \Lambda\right) \times g_{\mu\nu} \right\rangle$
		$\left\langle G_{\mu\nu} \equiv \left(\frac{R}{D} - \frac{R}{2}\right) \times g_{\mu\nu} \right\rangle$	$\left\langle \frac{R}{2} \times g_{\mu\nu} \right\rangle$	$\left\langle R_{\mu\nu} \equiv \frac{R}{D} \times g_{\mu\nu} \right\rangle$

Table 16. Either momentum **or** curvature

In the last consequence, either momentum or curvature conditions are based on the relationship

$$G_{\mu\nu} \equiv \left(\frac{R}{D} - \frac{R}{2}\right) \times g_{\mu\nu} \equiv \frac{R}{2} \times g_{\mu\nu} - \Lambda \times g_{\mu\nu} \tag{760}$$

and at the end demand that

$$\left(\frac{R}{D} \times g_{\mu\nu}\right) \equiv \left(R \times g_{\mu\nu}\right) - \left(\Lambda \times g_{\mu\nu}\right) \tag{761}$$

Other conditions can be found in literature(Barukčić, 2016b).

Theorem 3.59 (Four basic fields of nature)**.**

What are the four fundamental forces of objective reality, or of nature as such? Physics has already identified different types of interaction like strong nuclear force, weak nuclear force, electromagnetism and gravitation. However, the outstanding question is whether the fundamental forces of nature can be merged with the others. In this context, we redefine again the following,

		Curvature YES	Curvature NO	
Momentum	YES	$a_{\mu\nu}$	$b_{\mu\nu}$	$\left(\frac{R}{D}-\frac{R}{2}+\Lambda\right)\times g_{\mu\nu} \equiv \frac{8\times\pi\times\gamma}{c^4\times D}\times g_{\mu\nu}$
	NO	$c_{\mu\nu}$	$d_{\mu\nu}$	$\left(\frac{R}{2}-\Lambda\right)\times g_{\mu\nu}$
		$G_{\mu\nu}\equiv\left(\frac{R}{D}-\frac{R}{2}\right)\times g_{\mu\nu}$	$\frac{R}{2}\times g_{\mu\nu}$	$R_{\mu\nu}\equiv\frac{R}{D}\times g_{\mu\nu}$

Table 17. Fundamental fields of nature.

where $a_{\mu\nu}$ is the stress-energy tensor of the ordinary matter, $b_{\mu\nu}$ is the stress-energy tensor of the electro-magnetic field, $c_{\mu\nu}$ is the tensor of gravitation, $d_{\mu\nu}$ is at this moment an unknown tensor, $G_{\mu\nu}$ is the Einstein tensor, R is Ricci scalar.

Proof by direct proof. In general, the Einstein field equation demand that

$$\underbrace{G_{\mu\nu}+(\Lambda\times g_{\mu\nu})}_{left-hand\ side}\equiv\underbrace{\left(\frac{4\times 2\times\pi\times\gamma}{c^4}\right)\times T_{\mu\nu}}_{right-hand\ side} \equiv a_{\mu\nu}+b_{\mu\nu} \tag{762}$$

The left-hand side of equation 762 contains already its own the two determining fields, the tensor of the ordinary matter ($a_{\mu\nu}$) and the electromagnetic field ($b_{\mu\nu}$), whatever manipulations might be done with the left-hand side of the Einstein field equations (see equation 762). It is possible to specify the left-hand side of the Einstein field equation more precisely. The the tensor of the ordinary matter ($a_{\mu\nu}$) is already identified as

$$a_{\mu\nu}\equiv G_{\mu\nu}-x_{\mu\nu} \tag{763}$$

For various reasons not set out, the tensor $x_{\mu\nu}$ may stay preliminary, unknown even if the tensor $x_{\mu\nu}$ is already identified (see Barukčić, 2020a,c). Focussed properly, the Einstein tensor is for sure one on determining part of the tensor of the ordinary matter ($a_{\mu\nu}$). The stress-energy tensor of the electromagnetic field ($b_{\mu\nu}$) is already identified (see Barukčić, 2020a,c) as

$$b_{\mu\nu} \equiv x_{\mu\nu} + (\Lambda \times g_{\mu\nu}) \tag{764}$$

To put it in a nutshell: the term $(\Lambda \times g_{\mu\nu})$ is certainly one determining part of the stress-energy tensor of the electromagnetic field ($b_{\mu\nu}$). Einstein field equations (see equation 762) changes slightly. It is

$$\begin{aligned}\underbrace{(G_{\mu\nu} - x_{\mu\nu})}_{oridnary\ matter} + \underbrace{(x_{\mu\nu} + (\Lambda \times g_{\mu\nu}))}_{electromagnetic\ field} &\equiv \underbrace{\left(\frac{4 \times 2 \times \pi \times \gamma}{c^4}\right) \times T_{\mu\nu}}_{right-hand\ side}\\ &\equiv a_{\mu\nu} + b_{\mu\nu}\end{aligned} \tag{765}$$

The structure of the tensor $x_{\mu\nu}$ need to be identified. The relationships of equation 765 are illustrated in more detail by table 18.

		Curvature		
		YES	NO	
Momentum	YES	$G_{\mu\nu}$ - $x_{\mu\nu}$	$x_{\mu\nu}$ + $(\Lambda \times g_{\mu\nu})$	$\left(\frac{R}{D} - \frac{R}{2} + \Lambda\right) \times g_{\mu\nu} \equiv \frac{8 \times \pi \times \gamma}{c^4 \times D} \times g_{\mu\nu}$
	NO	?	?	$\left(\frac{R}{2} - \Lambda\right) \times g_{\mu\nu}$
		$G_{\mu\nu} \equiv \left(\frac{R}{D} - \frac{R}{2}\right) \times g_{\mu\nu}$	$\frac{R}{2} \times g_{\mu\nu}$	$R_{\mu\nu} \equiv \frac{R}{D} \times g_{\mu\nu}$

Table 18. Fundamental fields of nature in more detail.

However, the structure of neither the field $x_{\mu\nu}$ nor of the field $c_{\mu\nu}$ nor of the field $d_{\mu\nu}$ is identified (see table 17, page 138). As proofed somewhere else before (see Barukčić, 2016a,b, 2020a,b,c), it has to be that

$$\begin{aligned}\frac{R}{2} \times g_{\mu\nu} &\equiv b_{\mu\nu} + d_{\mu\nu}\\ &\equiv (x_{\mu\nu} + (\Lambda \times g_{\mu\nu})) + d_{\mu\nu}\end{aligned} \tag{766}$$

Briefly, and in plain words, the addition of the tensors $b_{\mu\nu}$ + $d_{\mu\nu}$ need to assure that the term $(\Lambda \times g_{\mu\nu})$ vanish. Consequently, one determining part of the tensor $d_{\mu\nu}$ is the term $(-\Lambda \times g_{\mu\nu})$ while another and unknown tensor $y_{\mu\nu}$ might remain. We obtain

$$\begin{aligned}d_{\mu\nu} &\equiv \frac{R}{2} \times g_{\mu\nu} - b_{\mu\nu}\\ &\equiv \left(\frac{R}{2} \times g_{\mu\nu} - x_{\mu\nu}\right) - (\Lambda \times g_{\mu\nu})\end{aligned} \tag{767}$$

		Curvature YES	NO	
Momentum	YES	$G_{\mu\nu} - x_{\mu\nu}$	$x_{\mu\nu} + (\Lambda \times g_{\mu\nu})$	$\left(\frac{R}{D} - \frac{R}{2} + \Lambda\right) \times g_{\mu\nu} \equiv \frac{8 \times \pi \times \gamma}{c^4 \times D} \times g_{\mu\nu}$
	NO	?	$\left(\frac{R}{2} \times g_{\mu\nu}\right) - x_{\mu\nu} - (\Lambda \times g_{\mu\nu})$	$\left(\frac{R}{2} - \Lambda\right) \times g_{\mu\nu}$
		$G_{\mu\nu} \equiv \left(\frac{R}{D} - \frac{R}{2}\right) \times g_{\mu\nu}$	$\frac{R}{2} \times g_{\mu\nu}$	$R_{\mu\nu} \equiv \frac{R}{D} \times g_{\mu\nu}$

Table 19. Fundamental fields of nature in more detail from another point of view.

More and more unknown tensors are necessary to solve the problems. The relationships as outlined just before are illustrated in more detail by table 19.

In general, it is

$$\begin{aligned}\left(\frac{R}{2} \times g_{\mu\nu}\right) - (\Lambda \times g_{\mu\nu}) &\equiv c_{\mu\nu} + d_{\mu\nu} \\ &\equiv c_{\mu\nu} + \left(\frac{R}{2} \times g_{\mu\nu} - x_{\mu\nu}\right) - (\Lambda \times g_{\mu\nu})\end{aligned} \tag{768}$$

Equation 768 simplifies as

$$0 \equiv c_{\mu\nu} - x_{\mu\nu} \tag{769}$$

At the end, it has to be that,

$$c_{\mu\nu} \equiv x_{\mu\nu} \tag{770}$$

The final picture of the four basic fields of nature is pictured by table 20.

		Curvature YES	NO	
Momentum	YES	$G_{\mu\nu} - x_{\mu\nu}$	$x_{\mu\nu} + (\Lambda \times g_{\mu\nu})$	$\left(\frac{R}{D} - \frac{R}{2} + \Lambda\right) \times g_{\mu\nu} \equiv \frac{8 \times \pi \times \gamma}{c^4 \times D} \times g_{\mu\nu}$
	NO	$x_{\mu\nu}$	$\left(\frac{R}{2} \times g_{\mu\nu}\right) - x_{\mu\nu} - (\Lambda \times g_{\mu\nu})$	$\left(\frac{R}{2} - \Lambda\right) \times g_{\mu\nu}$
		$G_{\mu\nu} \equiv \left(\frac{R}{D} - \frac{R}{2}\right) \times g_{\mu\nu}$	$\frac{R}{2} \times g_{\mu\nu}$	$R_{\mu\nu} \equiv \frac{R}{D} \times g_{\mu\nu}$

Table 20. Fundamental fields of nature in more detail from another point of view.

One exact solution of the Einstein field equations is the condition that

$$\frac{R}{2} \times g_{\mu\nu} \equiv x_{\mu\nu} \tag{771}$$

Under conditions of 771, equation 767 becomes

$$
\begin{aligned}
d_{\mu\nu} &\equiv \frac{R}{2} \times g_{\mu\nu} - b_{\mu\nu} \\
&\equiv \left(\frac{R}{2} \times g_{\mu\nu} - \frac{R}{2} \times g_{\mu\nu}\right) - \left(\Lambda \times g_{\mu\nu}\right) \\
&\equiv - \left(\Lambda \times g_{\mu\nu}\right)
\end{aligned}
\tag{772}
$$

Under these circumstances (see equation 772) the four basic fields of nature were determined, as illustrated by table 21.

		Curvature YES	NO	
Momentum	YES	$\left(\frac{R}{D} - R\right) \times g_{\mu\nu}$	$\left(\frac{R}{2} + \Lambda\right) \times g_{\mu\nu}$	$\left(\frac{R}{D} - \frac{R}{2} + \Lambda\right) \times g_{\mu\nu}$
	NO	$\frac{R}{2} \times g_{\mu\nu}$	- $\left(\Lambda \times g_{\mu\nu}\right)$	$\left(\frac{R}{2} - \Lambda\right) \times g_{\mu\nu}$
		$\left(\frac{R}{D} - \frac{R}{2}\right) \times g_{\mu\nu}$	$\frac{R}{2} \times g_{\mu\nu}$	$\frac{R}{D} \times g_{\mu\nu}$

Table 21. Fundamental fields of nature under conditions where $-\left(\Lambda \times g_{\mu\nu}\right)$ is an own fundamental field of nature.

There are circumstances where one basic field of nature is determined by the relationship

$$
d_{\mu\nu} \equiv - \left(\Lambda \times g_{\mu\nu}\right) \tag{773}
$$

One aim of the theorem before is to equip researchers with different points of view which are necessary to focus in more detail on the systematic formulation of the research question, what is the geometrical structure of the four basic fields of nature. We can't avoid recognizing again and again that there might exist circumstances or manifolds which are based on the relationship

$$
d_{\mu\nu} \equiv - \left(\Lambda \times g_{\mu\nu}\right) \tag{774}
$$

□

Table 22. The four basic fields of nature and general theory of relativity.

		Curvature YES	Curvature NO	
Momentum	YES	$\left(\frac{R}{D}-\frac{R}{2}+\Lambda\right)\times g_{\mu\nu}-\left(\frac{1}{4\times\pi}\times\left(\left(F_{\mu\,\mathrm{c}}\times F_{\nu}{}^{\mathrm{c}}\right)+\left(\frac{1}{4}\times g_{\mu\nu}\times F_{\mathrm{de}}\times F^{\mathrm{de}}\right)\right)\right)$	$\left(\frac{1}{4\times\pi}\times\left(\left(F_{\mu\,\mathrm{c}}\times F_{\nu}{}^{\mathrm{c}}\right)+\left(\frac{1}{4}\times g_{\mu\nu}\times F_{\mathrm{de}}\times F^{\mathrm{de}}\right)\right)\right)$	$\left(\frac{R}{D}-\frac{R}{2}+\Lambda\right)\times g_{\mu\nu}$
	NO	$\left(\frac{1}{4\times\pi}\times\left(\left(F_{\mu\,\mathrm{c}}\times F_{\nu}{}^{\mathrm{c}}\right)+\left(\frac{1}{4}\times g_{\mu\nu}\times F_{\mathrm{de}}\times F^{\mathrm{de}}\right)\right)\right)-\left(\Lambda\times g_{\mu\nu}\right)$	$\left(\frac{R}{2}\times g_{\mu\nu}\right)-\left(\frac{1}{4\times\pi}\times\left(\left(F_{\mu\,\mathrm{c}}\times F_{\nu}{}^{\mathrm{c}}\right)+\left(\frac{1}{4}\times g_{\mu\nu}\times F_{\mathrm{de}}\times F^{\mathrm{de}}\right)\right)\right)$	$\left(\frac{R}{2}-\Lambda\right)\times g_{\mu\nu}$
		$\left(\frac{R}{D}-\frac{R}{2}\right)\times g_{\mu\nu}$	$\frac{R}{2}\times g_{\mu\nu}$	$\frac{R}{D}\times g_{\mu\nu}$

Theorem 3.60. *In general, it is*

$$\hbar \times \omega_{\mu\nu} \equiv h \times f_{\mu\nu} \tag{775}$$

Proof by direct proof. At the beginning of this theorem, it is necessary and appropriate that an important point is being made about the theoretical starting point. All the subsequent content of this theorem stems from premise (i.e. axiom)

$$+1 \equiv +1 \tag{776}$$

Rearranging this equation, we obtain the foundation of the Einstein field equations as

$$\frac{R}{D} - \left(\frac{R}{2}\right) + (\Lambda) \equiv \left(\frac{8 \times \pi \times \gamma \times T}{c^4 \times D}\right) \tag{777}$$

Eq. 777 can be rearranged as

$$\left(\frac{\hbar}{\hbar}\right) \times \left(\frac{R}{D} - \left(\frac{R}{2}\right) + (\Lambda)\right) \equiv \left(\frac{h}{h}\right) \times \left(\left(\frac{8 \times \pi \times \gamma \times T}{c^4 \times D}\right)\right) \tag{778}$$

In the following, eq. 778 can be rearranged as

$$(\hbar) \times \left(\left(\frac{R}{\hbar \times D}\right) - \left(\frac{R}{2 \times \hbar}\right) + \left(\frac{\Lambda}{\hbar}\right)\right) \equiv (h) \times \left(\frac{8 \times \pi \times \gamma \times T}{h \times c^4 \times D}\right) \tag{779}$$

Under conditions where $\omega \equiv \left(\left(\frac{R}{\hbar \times D}\right) - \left(\frac{R}{2 \times \hbar}\right) + \left(\frac{\Lambda}{\hbar}\right)\right)$ and $f \equiv \left(\frac{8 \times \pi \times \gamma \times T}{h \times c^4 \times D}\right)$ it is

$$\hbar \times \omega \equiv h \times f \tag{780}$$

Multiplying equation 780 by the metric tensor, it is

$$\hbar \times \omega \times g_{\mu\nu} \equiv h \times f \times g_{\mu\nu} \tag{781}$$

We define

$$\omega_{\mu\nu} \equiv \omega \times g_{\mu\nu} \tag{782}$$

and

$$f_{\mu\nu} \equiv \left(\frac{8 \times \pi \times \gamma \times T}{h \times c^4 \times D}\right) \times g_{\mu\nu} \tag{783}$$

The generally covariant Planck-Einstein relation (referred to as Planck's energy–frequency relation, the Planck relation, Planck equation or Planck formula) is given as

$$\hbar \times \omega_{\mu\nu} \equiv h \times f_{\mu\nu} \tag{784}$$

□

The definitions $\omega \equiv \left(\left(\frac{R}{\hbar \times D} \right) - \left(\frac{R}{2 \times \hbar} \right) + \left(\frac{\Lambda}{\hbar} \right) \right)$ and $f \equiv \left(\frac{8 \times \pi \times \gamma \times T}{h \times c^4 \times D} \right)$ or similar ones $\left(\omega^{+2} \equiv \left(\left(\frac{R}{\hbar \times D} \right) - \left(\frac{R}{2 \times \hbar} \right) + \left(\frac{\Lambda}{\hbar} \right) \right) and f^{+2} \equiv \left(\frac{8 \times \pi \times \gamma \times T}{h \times c^4 \times D} \right) \right)$ are based on the assumption, that Einstein's field equations are nothing more but wave equations and that there is no contradiction between quantum theory and relativity theory. Of course, it is in no way sufficient to clarify facts by pure definition. Further evidence, especially experimental evidence, is needed in this context and equally highly valuable.

4. Discussion

Questions about being, nothing and becoming dominated by a variety of different views with respect to nothing and its relation to becoming have been debated by philosophers and other scientist for more than two millennia but equally without a resolution in sight. A different, but especially interesting example how to approach to the problem of the logical relationship between theory and (empirical) evidence has been published by Einstein (see also Howard, 2005). However, with their subject-specific methodology and terminology, the philosophers explore from their specific philosophical point of view the same objective reality as the physicists do with their physics-specific terminology and methodology. The unity of nature demand to us that a contradiction in this context besides of unquestioned traditional views would be difficult to accept. Especially, it is not human mind and consciousness or the subject-specific methodology and terminology which decides how objective reality is and has to be. For our purposes, modern physics can enable us to generate new insights into those old questions. Until contrary proof, we assume the identity of **being = energy**, **nothing = time** and **becoming = space**. These distinctions might prove helpful in the subsequent discussion of these basic notions in modern science.

Albert Einstein introduced the cosmological term (Einstein, 1917) in his paper of 1917. Soon, the Dutch astronomer Willem De Sitter (1872–1934) discovered a matter-free anti-Machian cosmological model (De Sitter, 1917) which contained no matter but still incorporated Einstein's cosmological constant. Einstein himself was not very happy at that time with the logical possibility of supposing matter not to exist. Einstein wrote to de Sitter on March 24, 1917:

> "Es wäre nach meiner Meinung unbefriedigend, wenn es **eine** denkbare **Welt ohne Materie** gäbe.
>
> Das g-Feld *(gravitational field, author)* soll vielmehr
>
> *durch die Materie bedingt sein,*
>
> *ohne dieselbe nicht bestehen können.* "
>
> (see also De Sitter, 1917, p. 1125)

It is more than logical to ask the following question: if a world without matter/energy has existed or theoretically can exist while our world itself is full of matter/energy, where does this matter/energy come from? Does this mean that matter/energy can be created and if yes, out of what? In the end, do we have to accept the hypothesis of the creation of matter/energy ex nihilio as correct? What would the universe look like if a negative cosmological constant were an own fundamental field of nature? Unfortunately, this is not the place to debate these gigantic and the far-reaching issues. Even if space-time itself can be flat (Minkowski space (or space-time in 4 dimensions)) or non-flat (i.e. curved) et cetera, there is considerable doubt, however, about the adequacy of general validity of equation 774 as a fourth basic field of nature even if an n-dimensional anti-de Sitter spacetime, also termed de Sitter spacetime (Calabi and Markus, 1962) of the second kind, named after Willem de Sitter

(1872–1934), corresponds closely to a negative cosmological constant. Theoretically, it seems to me to be necessary to consider manifolds or circumstances where a negative cosmological constant is a basic field of nature. However, there remains an open question whether this is in general the case. If we took equation 773 to its logical conclusion, it would mean that the unity of gravitation and electromagnetism would be given as

$$b_{\mu\nu} + c_{\mu\nu} \equiv \left(R \times g_{\mu\nu}\right) + \left(\Lambda \times g_{\mu\nu}\right) \tag{785}$$

while

$$b_{\mu\nu} + c_{\mu\nu} + d_{\mu\nu} \equiv \left(R \times g_{\mu\nu}\right) \tag{786}$$

The four basic fields of nature are identified and illustrated in greater detail by table 22. Still, we have to leave the question open whether there might exist conditions where a negative cosmological constant $-\left(\Lambda \times g_{\mu\nu}\right)$ might break up into the parts $+\left(\frac{R}{2} \times g_{\mu\nu}\right)$ and $-\left(\frac{1}{4 \times \pi} \times \left(\left(F_{\mu\,\mathrm{c}} \times F_{\nu}{}^{\mathrm{c}}\right) + \left(\frac{1}{4} \times g_{\mu\nu} \times F_{\mathrm{de}} \times F^{\mathrm{de}}\right)\right)\right)$ as illustrated by equation 787.

$$-\left(\Lambda \times g_{\mu\nu}\right) \Longrightarrow +\left(\frac{R}{2} \times g_{\mu\nu}\right) - \left(\frac{1}{4 \times \pi} \times \left(\left(F_{\mu\,\mathrm{c}} \times F_{\nu}{}^{\mathrm{c}}\right) + \left(\frac{1}{4} \times g_{\mu\nu} \times F_{\mathrm{de}} \times F^{\mathrm{de}}\right)\right)\right) \tag{787}$$

Although, theoretically, we must bear in mind and take into consideration the possibility of a state of pure symmetry where

$$+\left(\Lambda \times g_{\mu\nu}\right) - \left(\Lambda \times g_{\mu\nu}\right) \equiv +0 \tag{788}$$

Equation 788 might be the natural foundation for something like spontaneous symmetry (Anderson, 1972) breaking and equally one condition for the beginning of our world. Can we escape from zero, under which conditions can we escape from zero, the state of pure symmetry, the black hole of mathematics?

5. Conclusion

Following the path of classical logic and relativity theory, a quantization of the gravitational field appears to be possible.

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6. Patient consent for publication

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Conflict of interest statement

No conflict of interest to declare.

Private note

The definition section of a paper need not and does not necessarily contain new scientific aspects. Above all, it also serves to better understand a scientific publication, to follow every step of the arguments of an author and to explain in greater details the fundamentals on which a publication is based. Therefore, there is no objective need to force authors to reinvent a scientific wheel once and again unless such a need appears obviously factually necessary. The effort to write about a certain subject in an original way in multiple publications does not exclude the necessity simply to cut and paste from an earlier work, and has nothing to do with self-plagiarism. However, such an attitude cannot simply be transferred to the sections' introduction, results, discussion and conclusions et cetera.

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